Accretion of Gas Giant Planets Constrained by the Tidal Barrier

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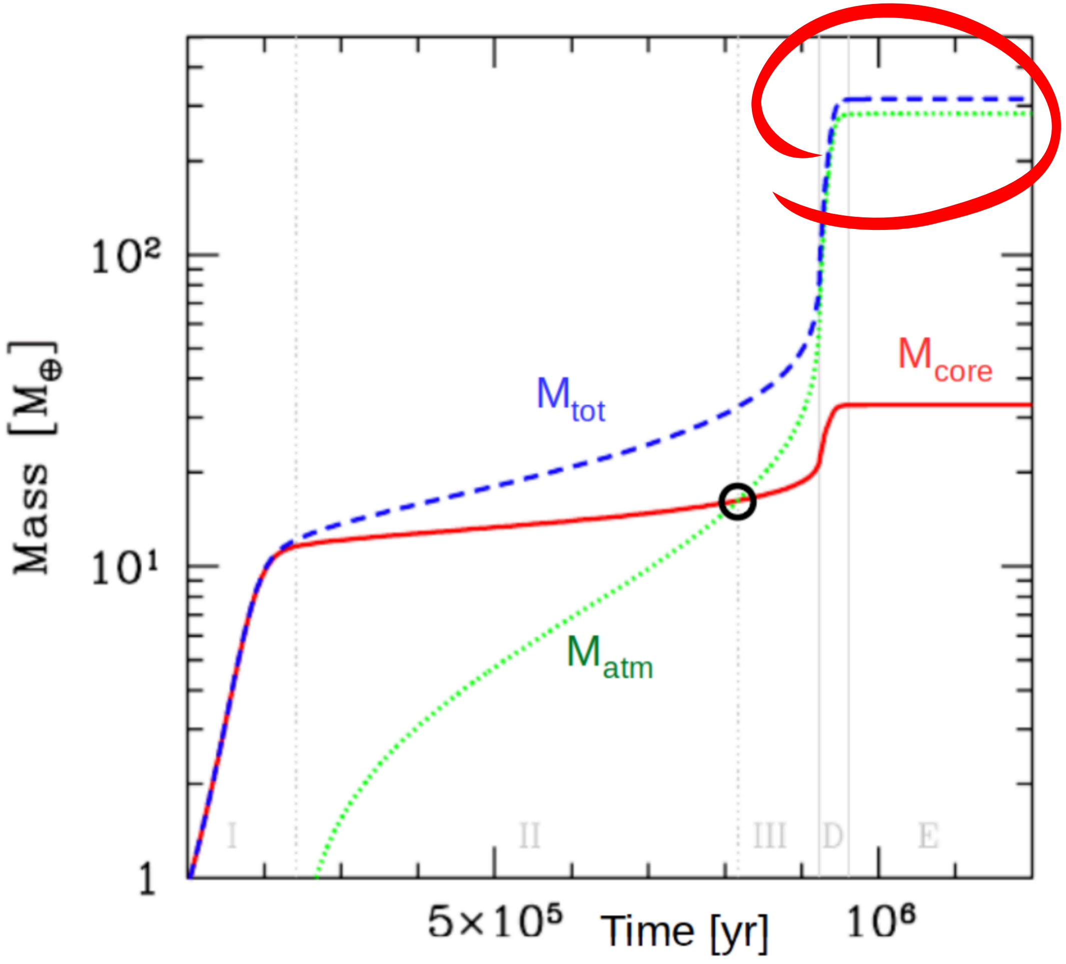
Outline

- Dynamical Accretion: Previous estimates and numerical results
- Effect of the Tidal Barrier: Shrinking the effective cross section
- Dynamical growth with large disk eccentricities
- Summary of numerical results of accretion rates
- Outstanding Issue: 3D & 2D discrepancy

Dynamical Accretion



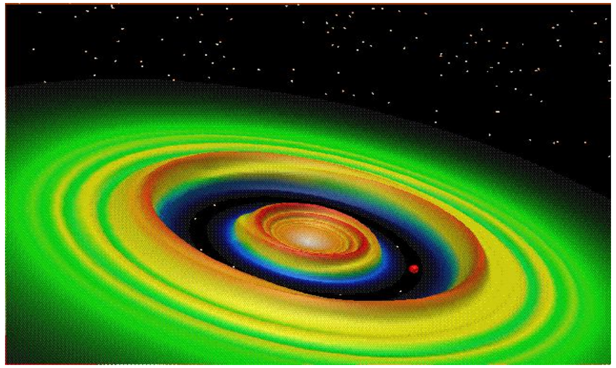
Mordasini et al. (2012)

What stops the last accretion stage of gas giants?

- Disk Depletion

but usually doubling timescale < Myr?

- Gap Opening $$R_H > H$$



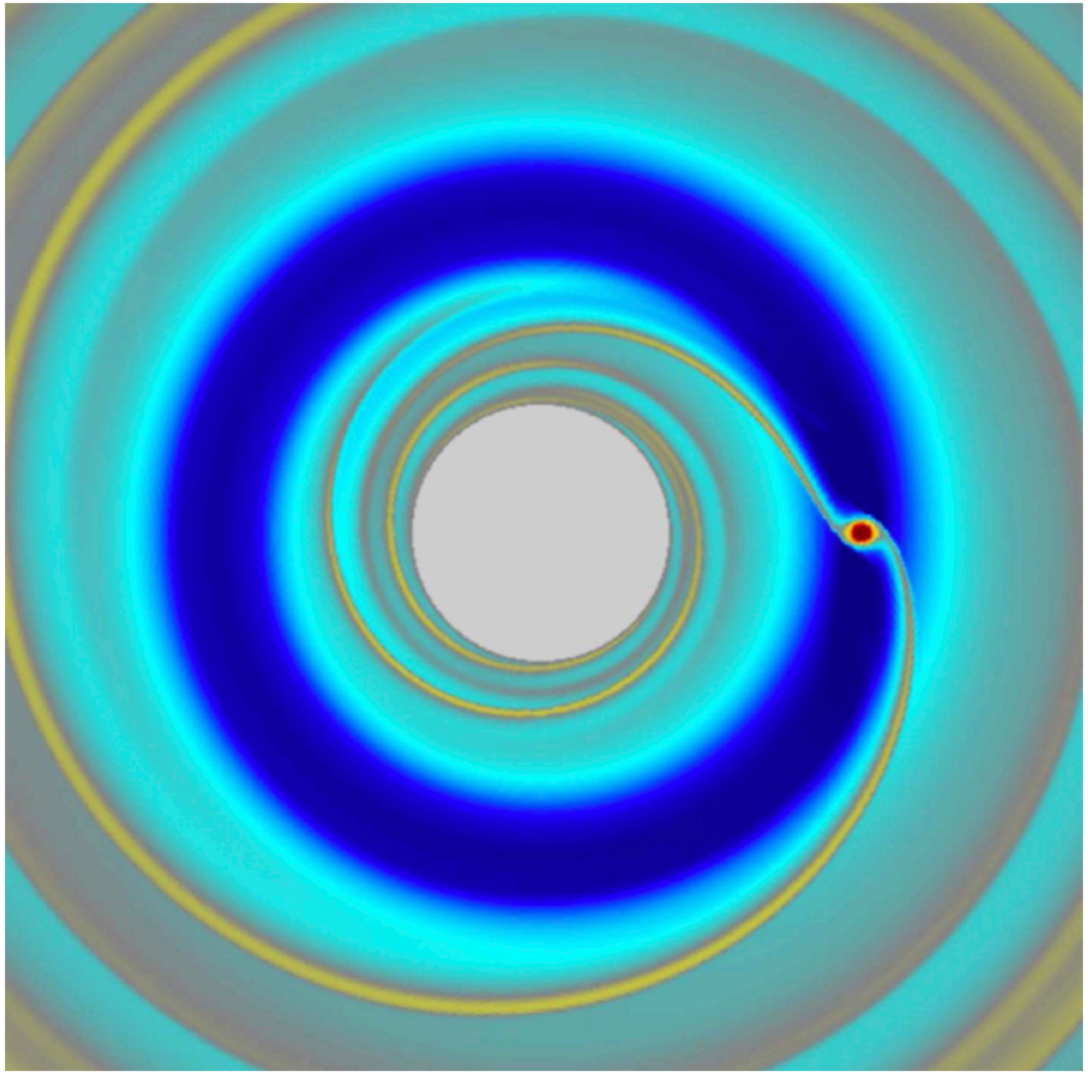
Bryden+ 2000

Constrains the planets to have masses of ~MJ

Previous estimates & Numerical results

However, recent simulations show the gap is never quite totally depleted!

In a typical gap carved out by a giant planet: materials can still flow inwards and is not cut off (e.g. Duffell+ 2014, Chen+ 2020)



Net *change* in disk surface density under planet perturbation

Previous estimates & Numerical results

In the new gap models, the gap maintains some non-zero bottom density.
(Duffell & MacFadyen 2013, Kanagawa+2015)

$$\frac{\Sigma_{\rm min}}{\Sigma_{\rm p}} \approx \frac{1}{1 + 0.04K}, \quad \text{where} \quad K \equiv q^2 h_{\rm p}^{-5} \alpha^{-1}.$$

This made the whole expression of the accretion rate as a function of planet mass possible

$$\dot{m}_{\rm p} = A\Sigma_{\rm min}.$$

Previous estimates & Numerical results

(5 AU, 1 orbit ~ 12 yrs)

Tanigawa & Watanabe 2002

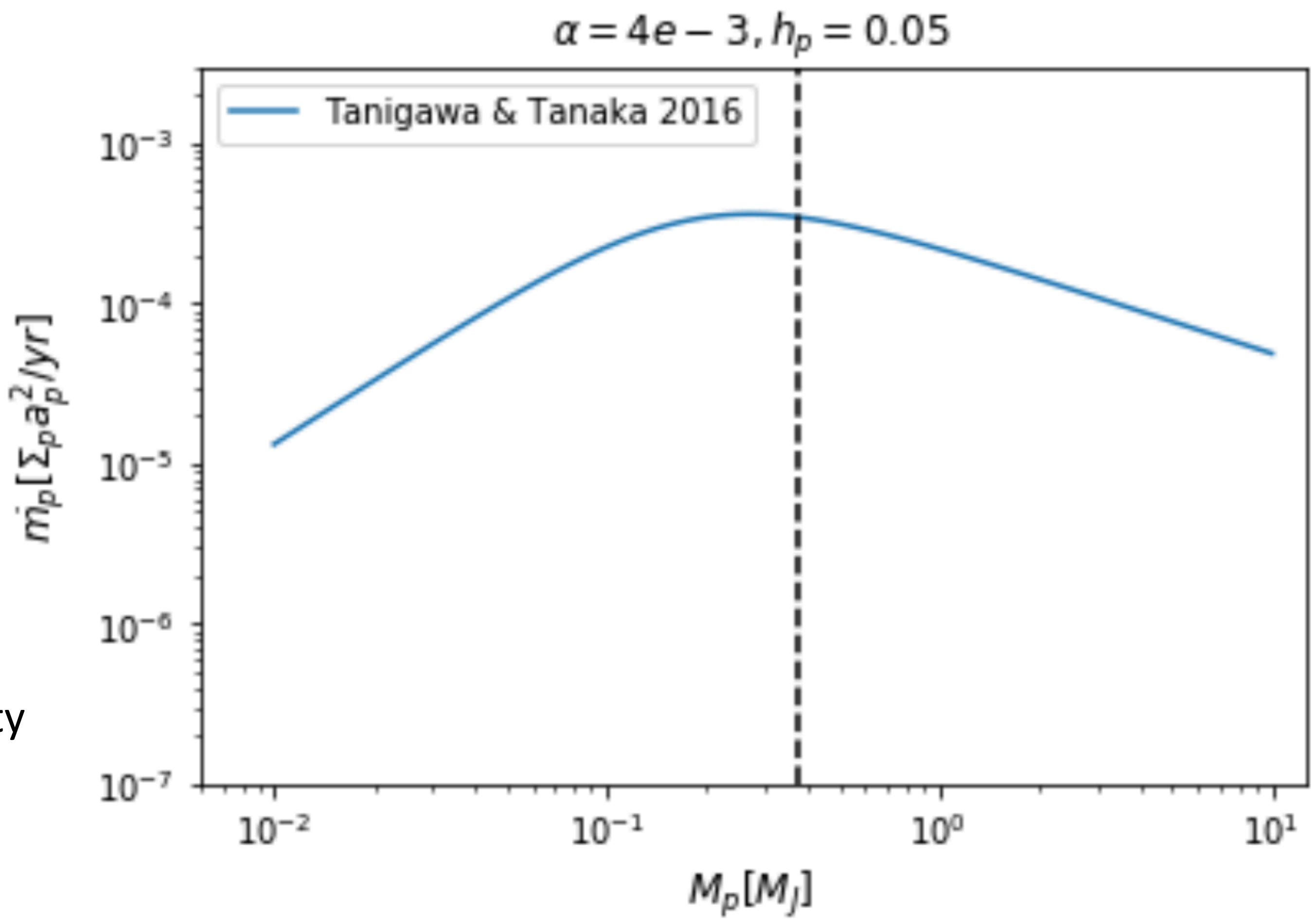
use similations to fit the relations

$$A = 0.29 h_p^{-2} q^{4/3} r_{\rm p}^2 \Omega_{\rm p}$$

$$\dot{m}_{\rm p} = A \Sigma_{\rm min}.$$

Grows as a power of 4/3 when surface density unperturbed

Then goes down due to gap-opening



Tanigawa & Tanaka 2016: the full prescription predicts typical gas giant mass of ~10 MJ!

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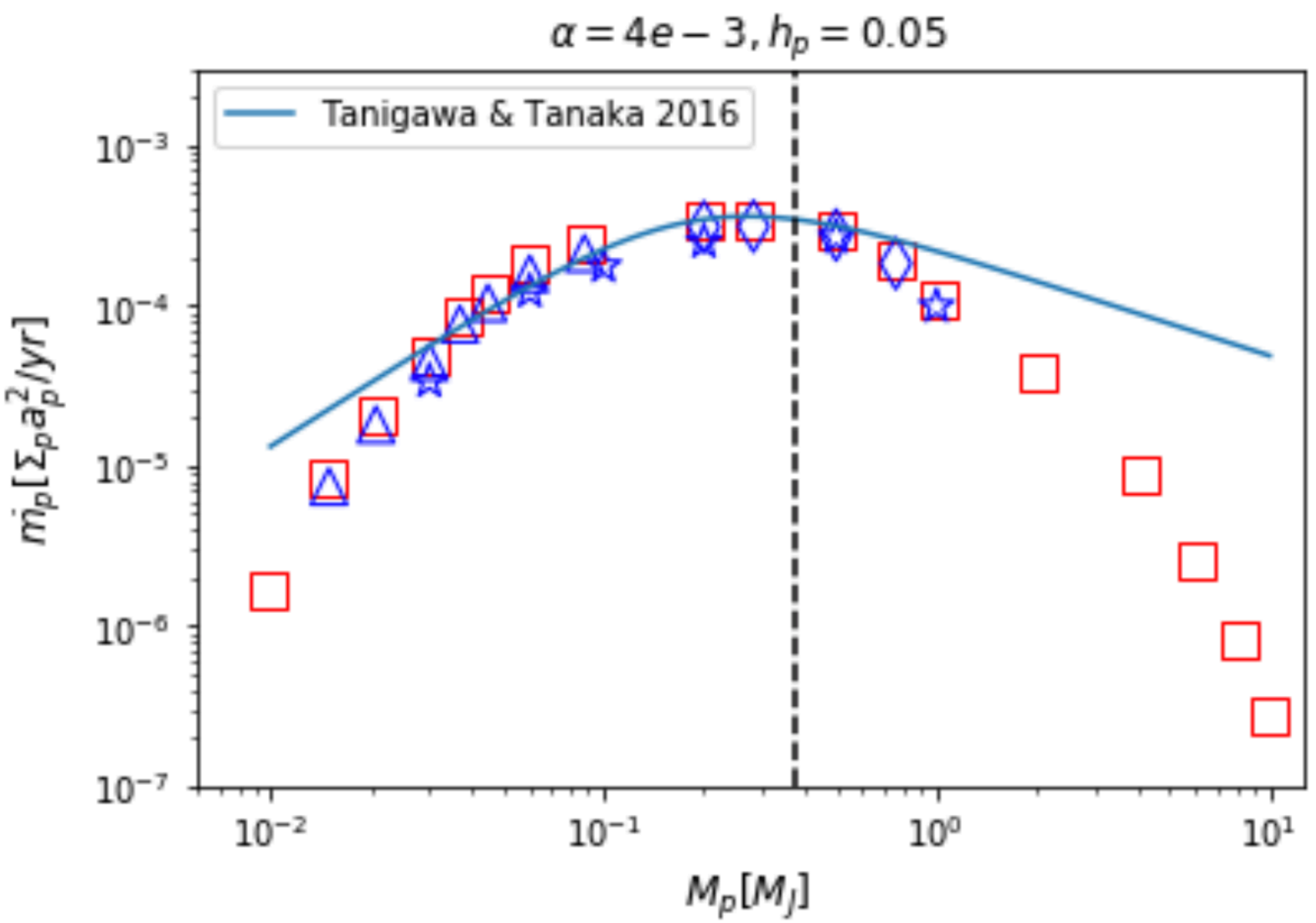
Compare with 3D nested-grid simulations?

D'Angelo+ 2003
Bodenheimer+ 2013

(5 AU, 1 orbit ~ 12 yrs)



Tanigawa & Tanaka 2016: the full prescription predicts typical gas giant mass of ~10 MJ!

Previous estimates & Numerical results

Rosenthal, Chiang + 2020

A simple analytical perspective:
accretion rate= flux * cross section

$$\rho v \sigma = \frac{\Sigma_{\rm min}}{H} v \sigma$$

Sub-thermal Bondi accretion

$$v \sim c_s, \sigma \sim R_B^2$$

$$\dot{m}_{\rm p} \sim \Sigma_{\rm min} c_{\rm s} R_{\rm B}^2 / H$$

Super-thermal Hill accretion

$$v \sim R_H \Omega_p, \sigma \sim R_H H$$

$$\dot{m}_{\rm p} \sim \Sigma_{\rm min} R_{\rm H}^2 \Omega_{\rm p}$$

Previous estimates & Numerical results

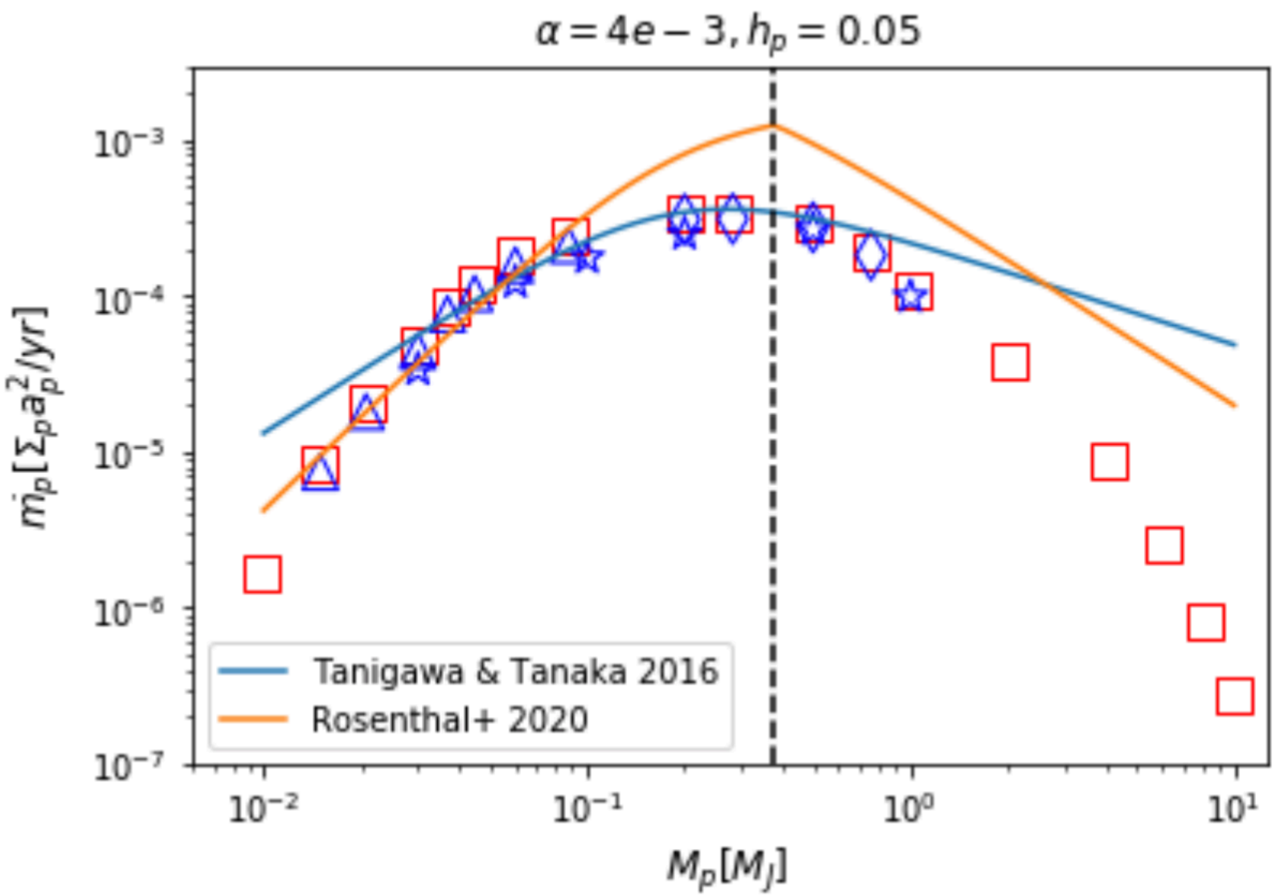
(5 AU, 1 orbit ~ 12 yrs)

$$A = \begin{cases} A_{\rm Bondi} = c_1 \frac{q^2}{h_{\rm p}^4} \Omega_{\rm p} a_{\rm p}^2 & q \le q_{\rm th} \\ A_{\rm Hill} = c_2 q^{2/3} \Omega_{\rm p} a_{\rm p}^2 & q > q_{\rm th} \end{cases}$$

Calibrated c1 and c2 by:

- Fitting c1 with D'Angelo+
- Requiring continuous transition at thermal mass

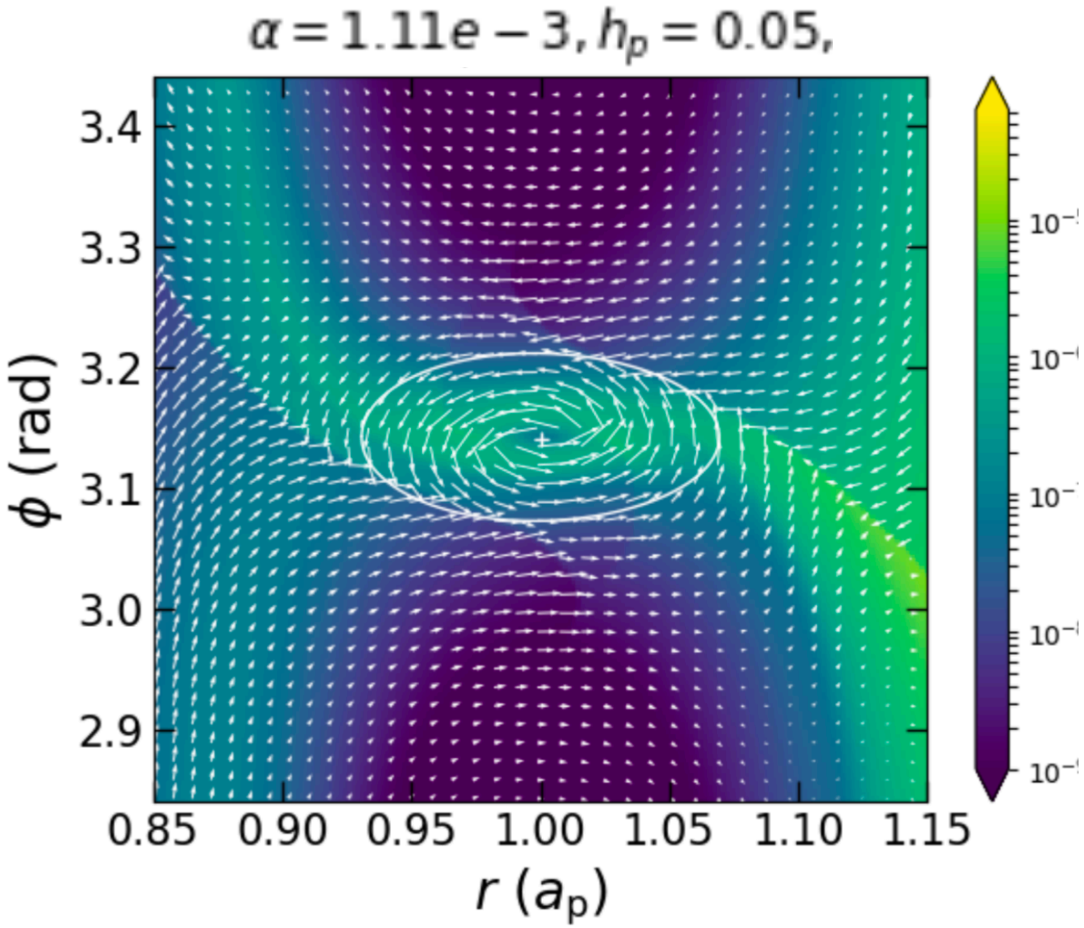
Still reproduces too much 10MJ planets!



Effect of the Tidal Barrier

Previous scalings neglect the tidal effects due to the non-axisymmetric potential in the proximity of the planets' Hills radius.

Is the effective cross section width always ~ R_H?



Effect of the Tidal Barrier

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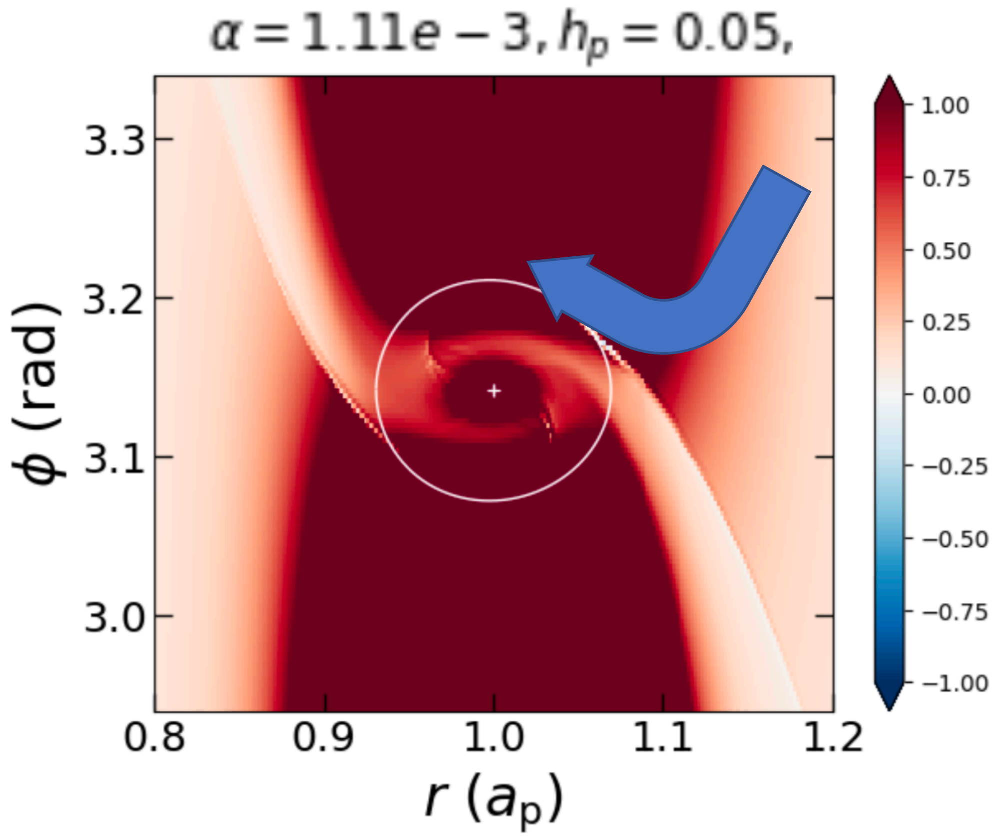
Dobbs-Dixon+ 2007:

By requiring Bernoulli energy & vortensity to conserve in the Roche potential field (inviscid limit):

$$A_{\rm Hill} \approx 2\pi R_{\rm H} H_{\rm p} \Omega_{\rm p} \exp\left[-\left(\frac{R_{\rm H}}{H_{\rm p}}\right)^2 - \frac{1}{2}\right]$$

vortensity $$\varpi = \frac{\omega + 2\Omega_{\rm p}}{\Sigma},$$

Effective width of the cross section <R_H!



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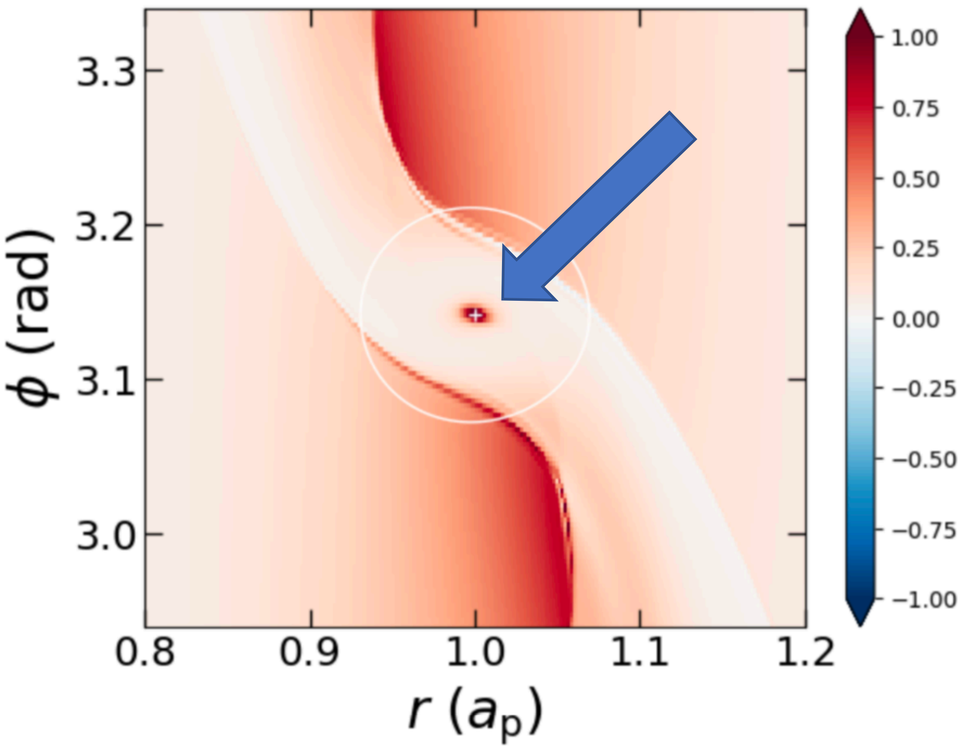
vortensity $$\varpi = \frac{\omega + 2\Omega_{\mathrm{p}}}{\Sigma},$$

But will be wiped out by viscosity

$\alpha = 1.11e-2, h_p = 0.05$



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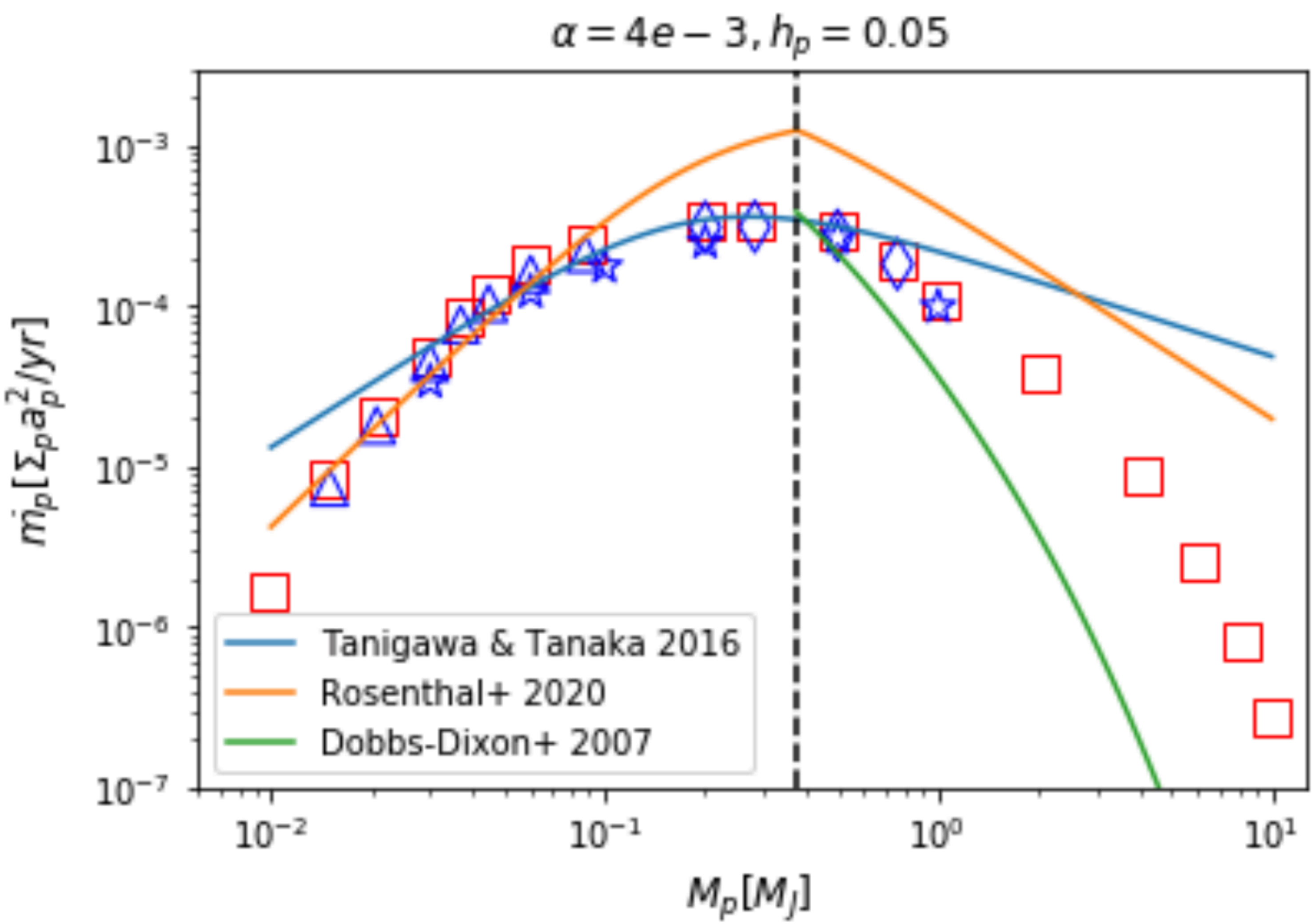


- DLL gives the lower bound is there is no viscosity and full conservation
- But the exponential decaying effect may still be important for moderate/intermediate viscosity scenarios!

Effect of the Tidal Barrier

That is just one parameter (and no uniform high resolution), We investigate

- two ends of the viscosity parameter alpha=1e-2 & 1e-3
- two scale heights h=0.03, 0.05
- a variety of super thermal planet mass

High resolution: 0.25 a_p - 8 a_p, 2048*2048

Fiducial disk:

$$T_{\rm disk} \propto r^{-\zeta}$$

$$\zeta = 0.5$$

$$\Sigma(r) = \Sigma_{\rm p} \left(\frac{r}{r_0}\right)^{-s},$$

$$s = 1.0$$

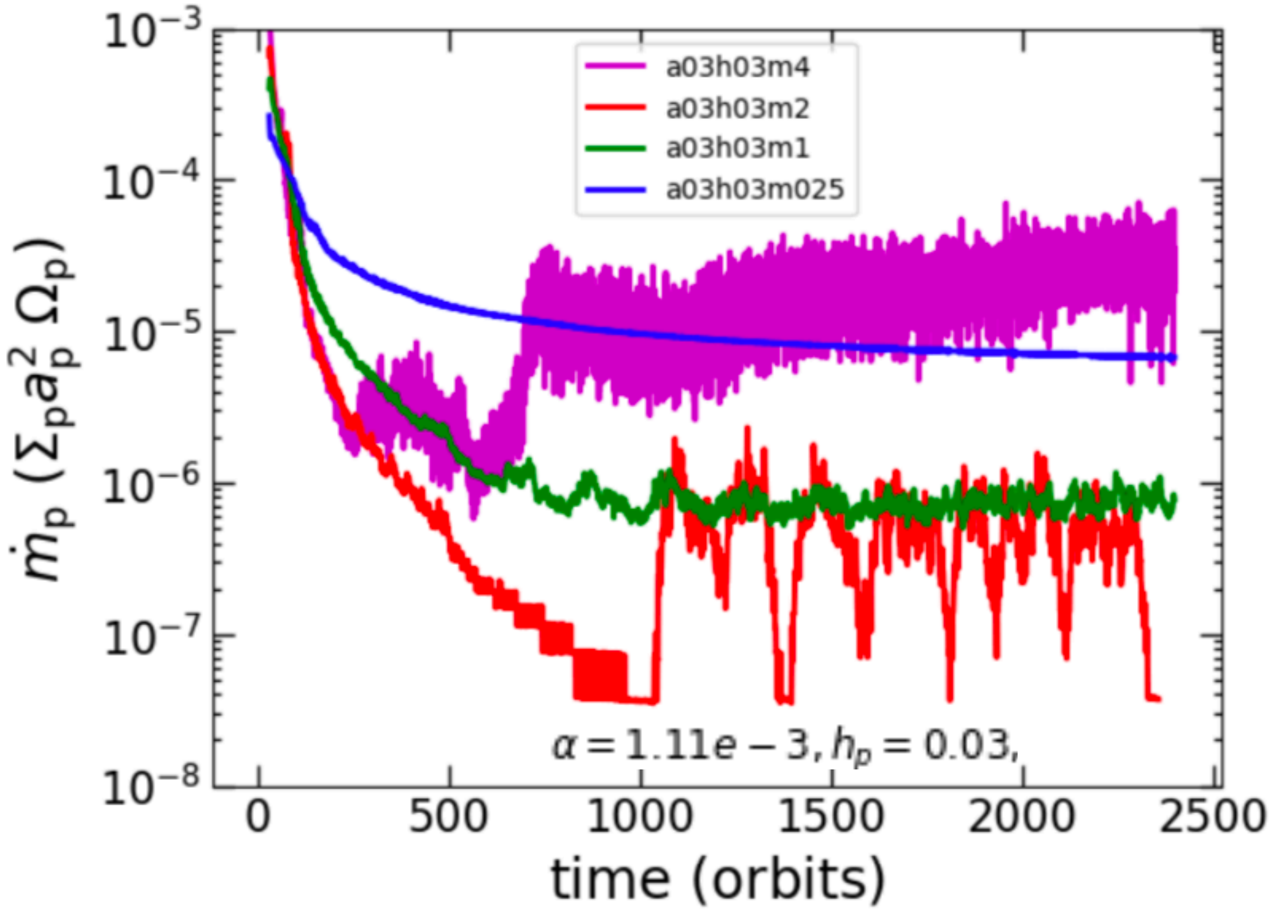
$$\dot{M}_* \sim 3\pi\alpha h_{\rm p}^2 a_{\rm p}^2 \Sigma_{\rm p} \Omega_{\rm p}$$

constant, and given by arbitrarily choosing Sigma_p

Fiducial Sigma_p is given by Sigma_p a_p^2=0.001

The Eccentricity Effect

Do a low-vis case, and we immediately see a problem...



The evolution of planetary accretion rate, measured in scale-free units,

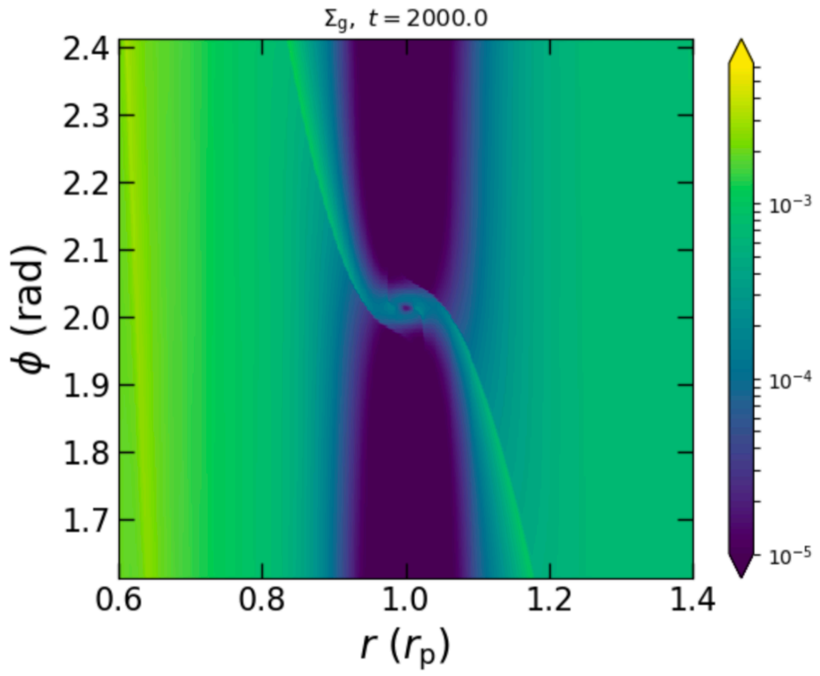
For large mass >=2MJ, the accretion rates decay rapidly in the first 1000 orbits. They then abruptly jump to much higher and more unstable values!

This is not surprising as it has been studied extensively before in 2D simulations (Kley & Dirksen 2005, Duffell & Chiang 2016), as a result of streamline eccentricity excitation.

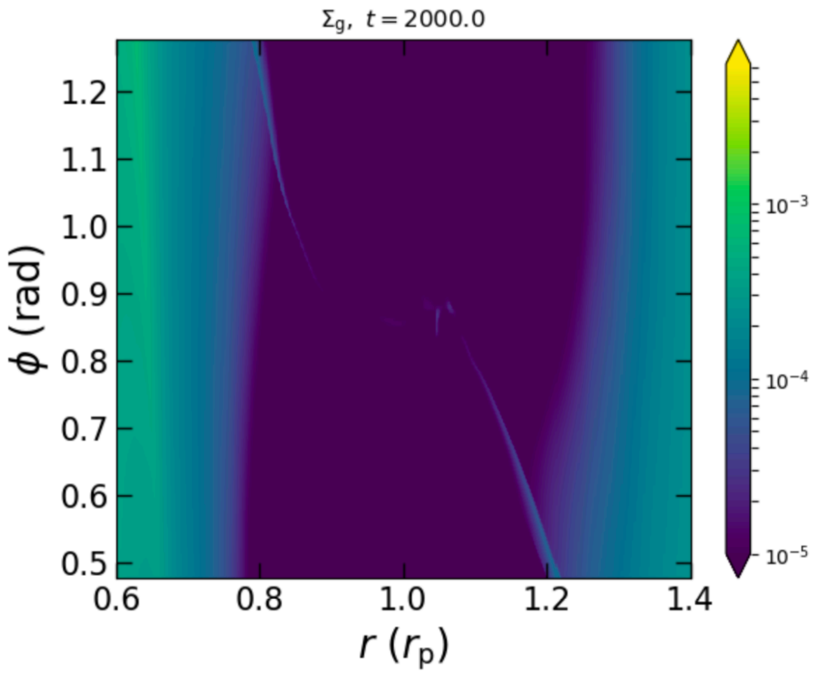
But reminds us that the runaway accretions might not be so “orderly” as predicted by any of the scalings!

The Eccentricity Effect

Although we fix the planet, eccentricity of the streamlines disrupts the conservation laws and the time-independence of orderly accretion



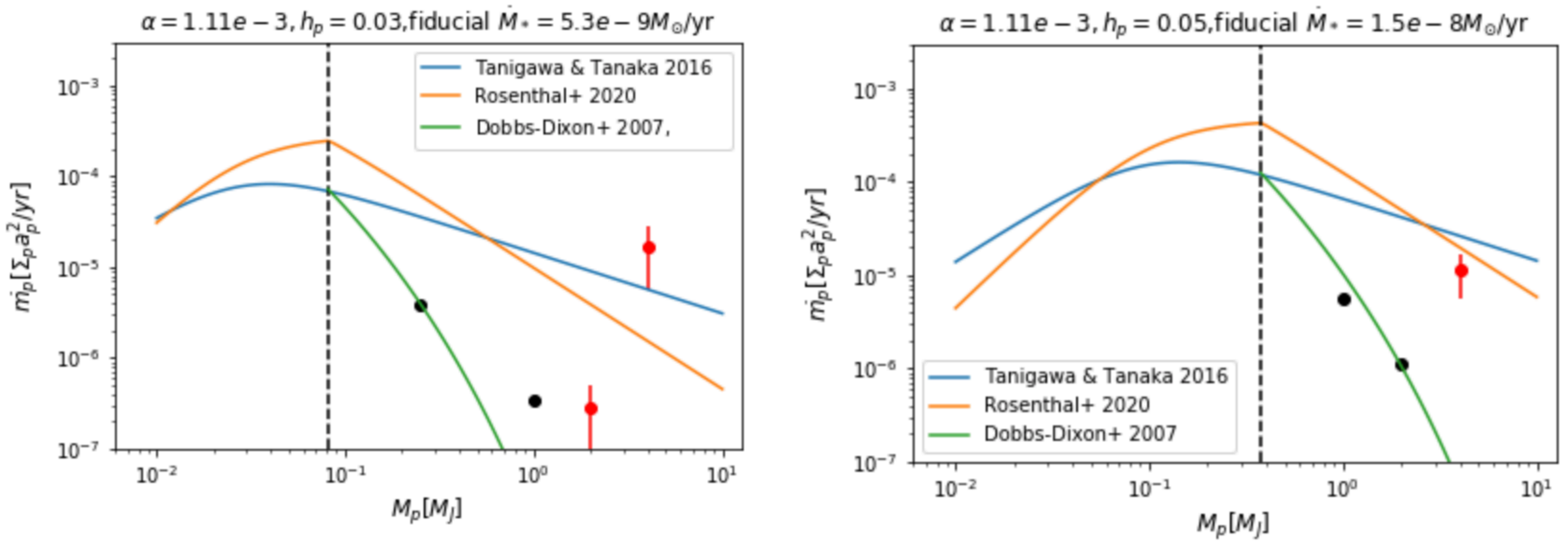
0.25 MJ



2 MJ

Unstable eccentricities are quantified verified in the cases of using Kley & Dirksen 2005 methods, but not the main focus of this paper

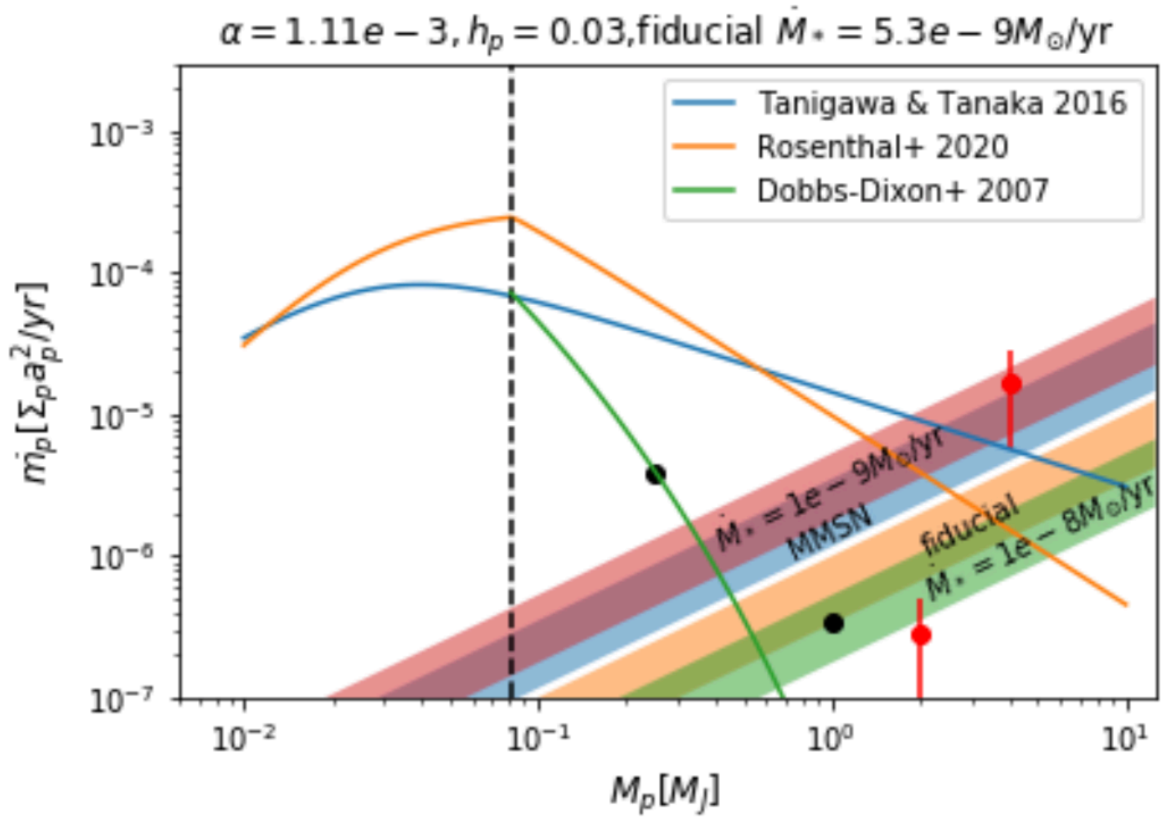
Summary of numerical results (low alpha)

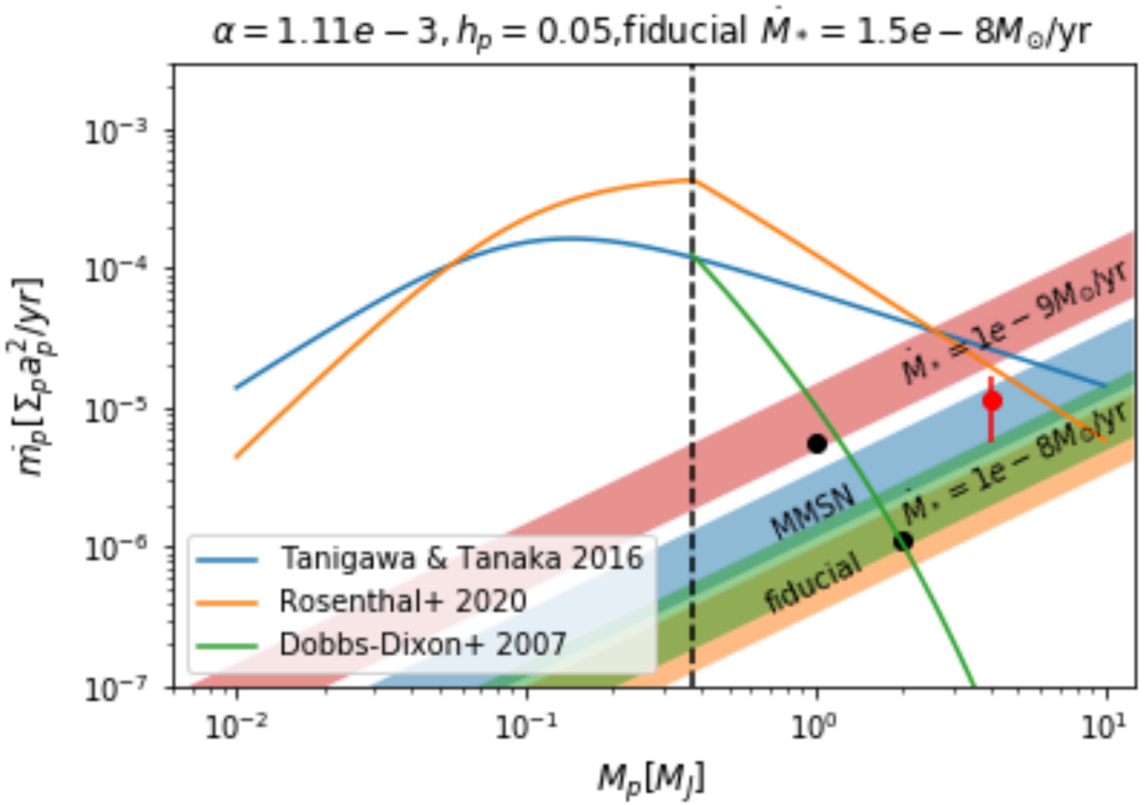


- Results for orderly accretion (before eccentricity excitation) agrees better with DLL scaling

- Although one fiducial surface density is used in simulation, can be extrapolated to any surface density/stellar accretion rate as long as no GI

Summary of numerical results (low alpha)





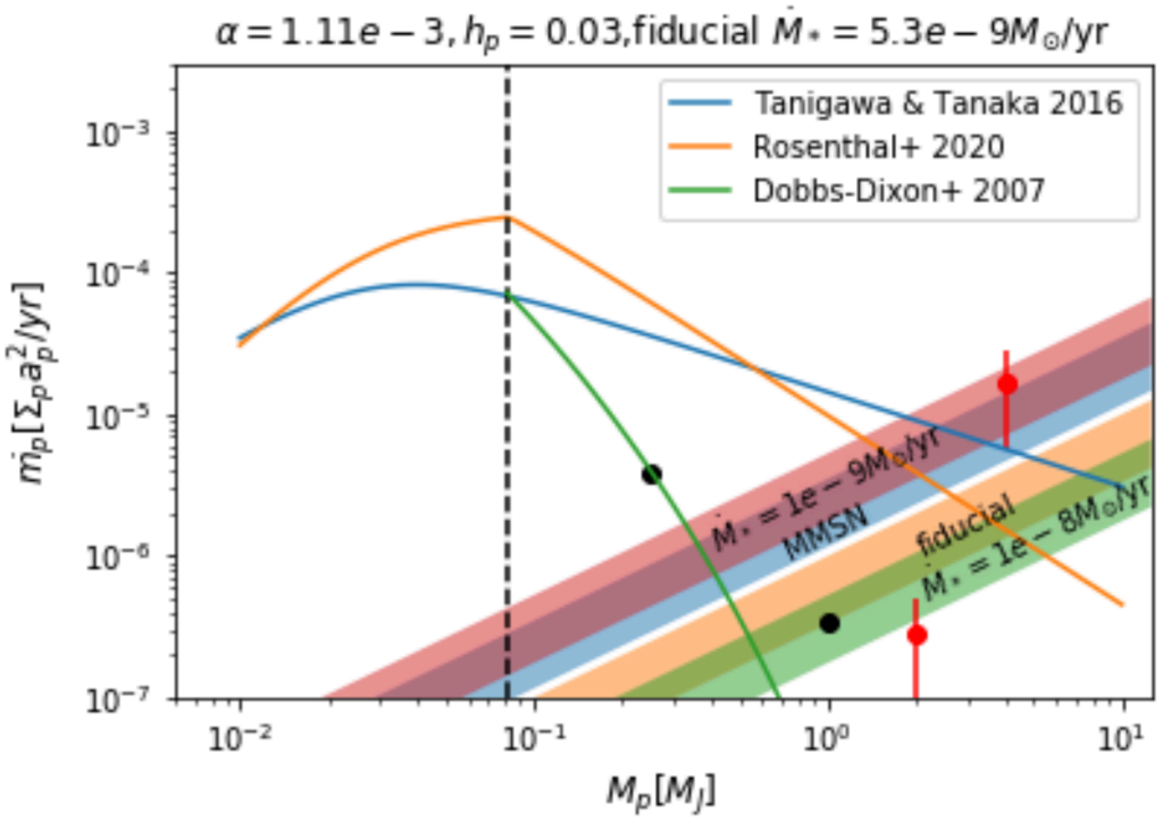
For a specified surface density/stellar accretion rate, we can determine the doubling timescale

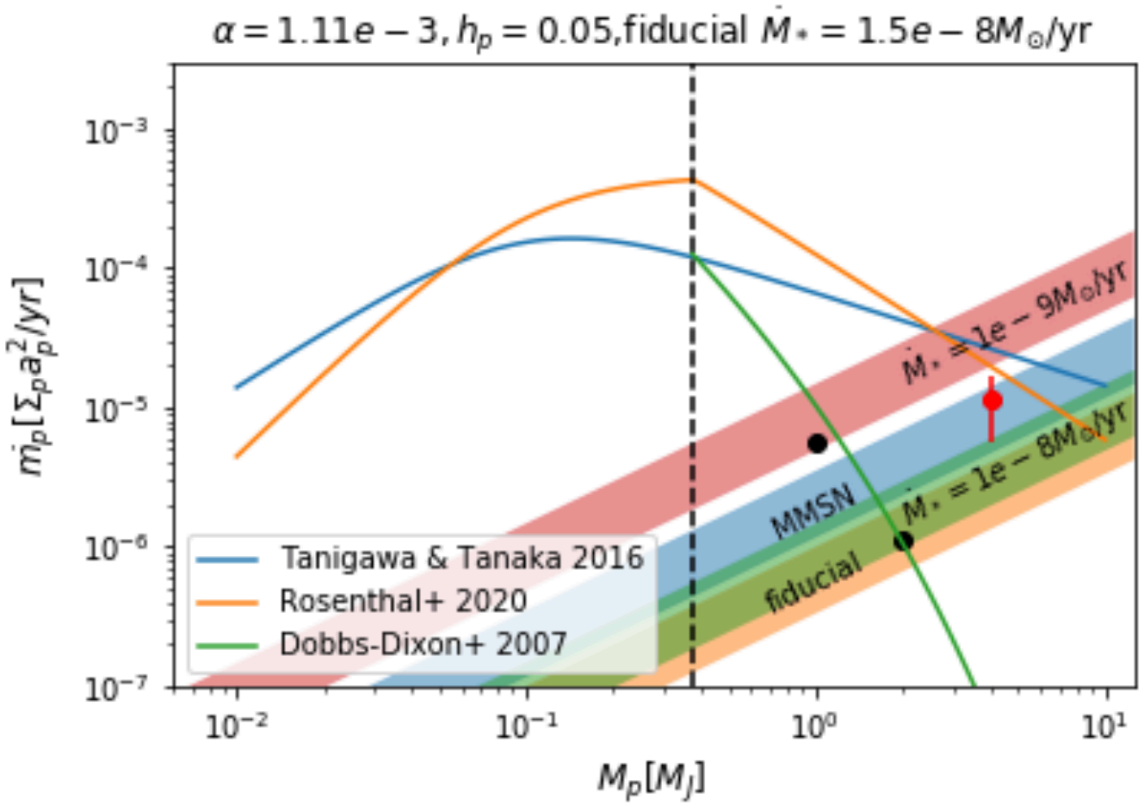
$$\dot{M}_* \sim 3\pi\nu\Sigma = 3\pi\alpha h_p^2 a_p^2 \Sigma_p \Omega_p$$

$$\tau_p = M_p / \dot{m}_p$$

Different color bands can indicate the parameter space for doubling time to be within 1-3 Myrs

Summary of numerical results (low alpha)

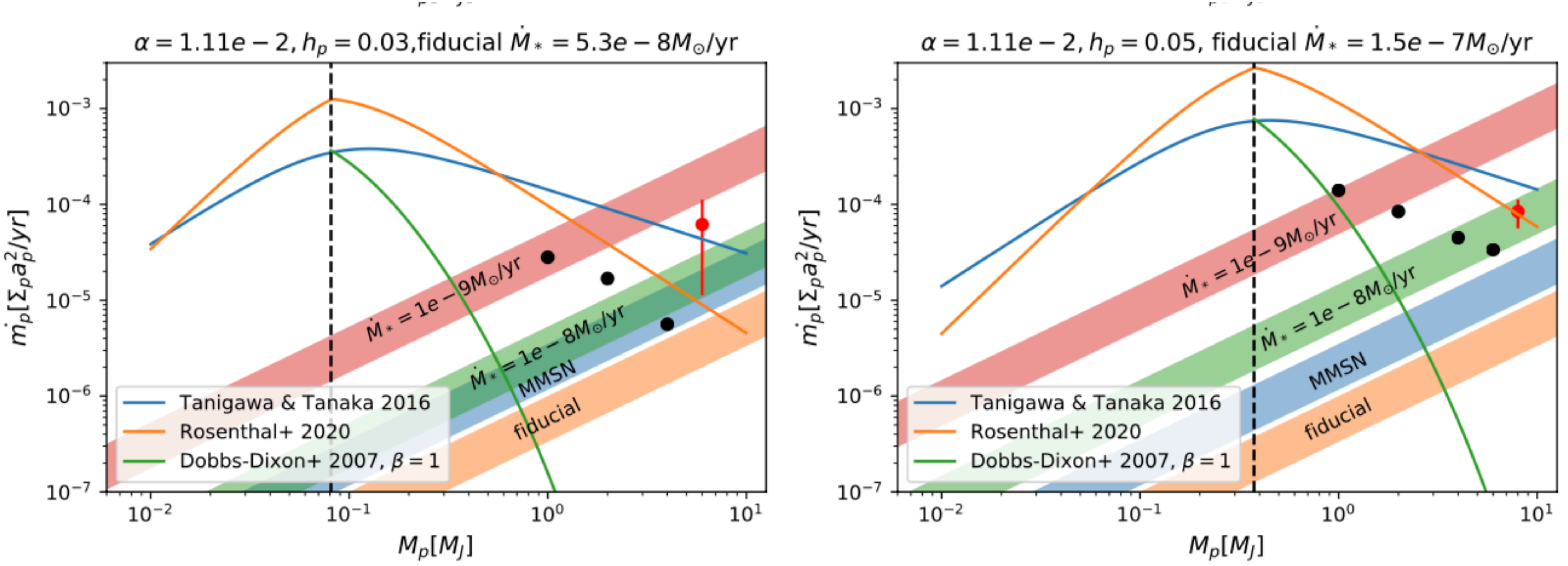




any dot within or below the bands indicate $\tau_p \gtrsim \tau_{dep}$

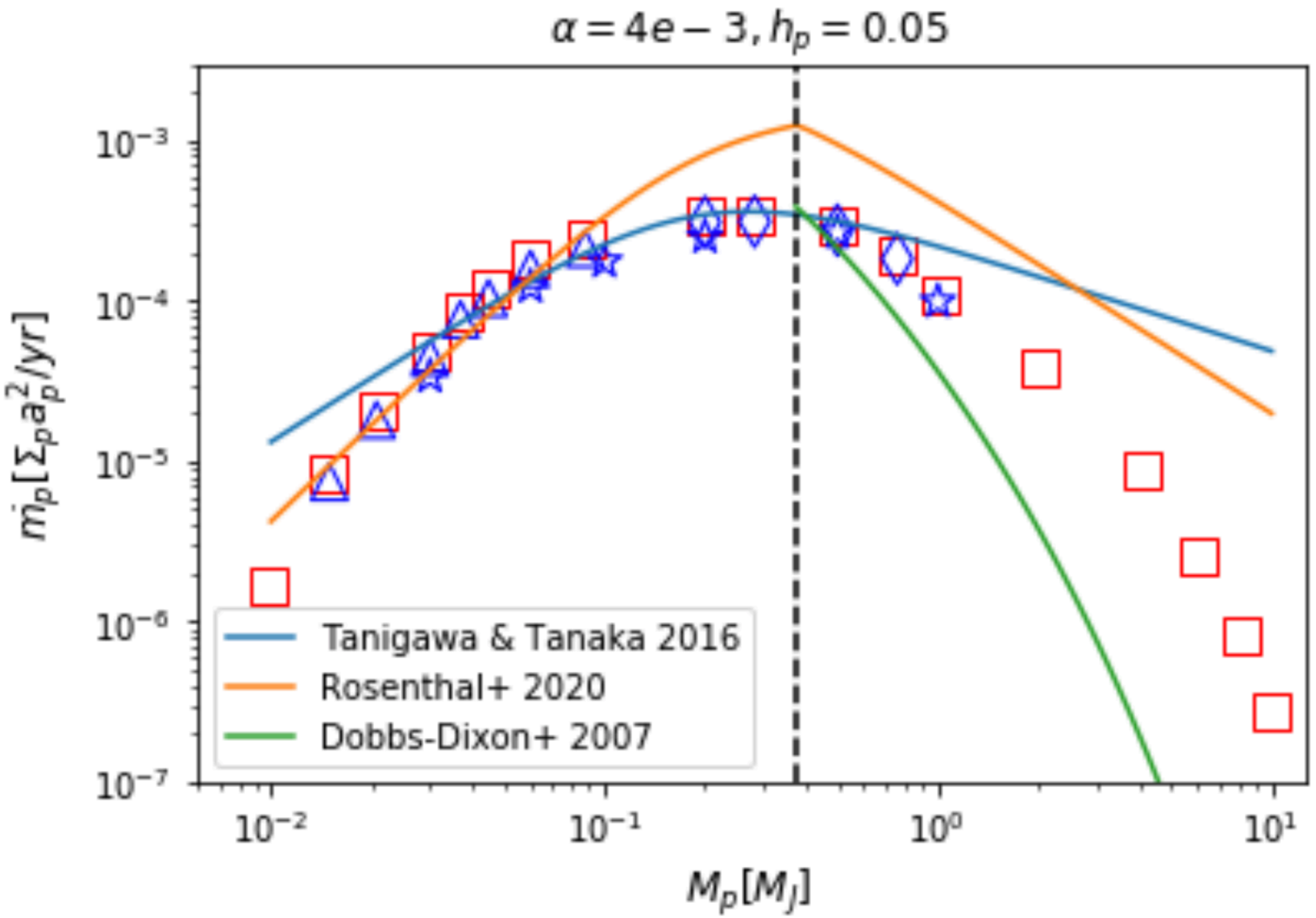
For such accretion rates, planets can only acquire modest masses prior to disk depletion in such environments, and unstable streamline eccentricity would not be excited in a self-consistent way.

Summary of numerical results (high alpha)



- High planetary accretion rates for the high- viscosity numerical models are in better agreement with the TT or RCGM scaling laws (that is, still before eccentricity excitation).
- Transition to unstable streamline eccentricity is likely to occur in high density environment, and further enhance the planets accretion rate, promote asymptotic masses to become much larger than that of Jupiter, unless in very evolved disks.
- Suggest typical Jupiter mass giants were born in disks with relatively low viscosity.

Outstanding Issue: 2D VS 3D

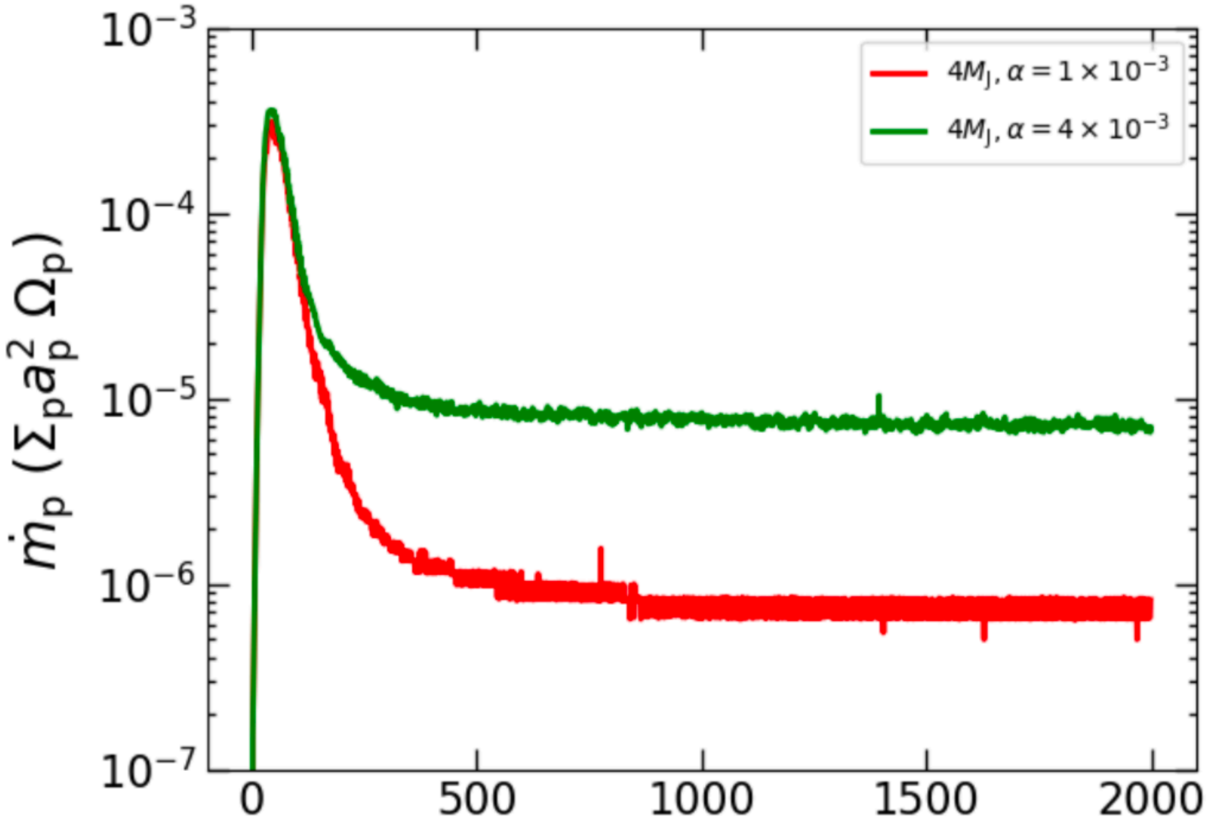


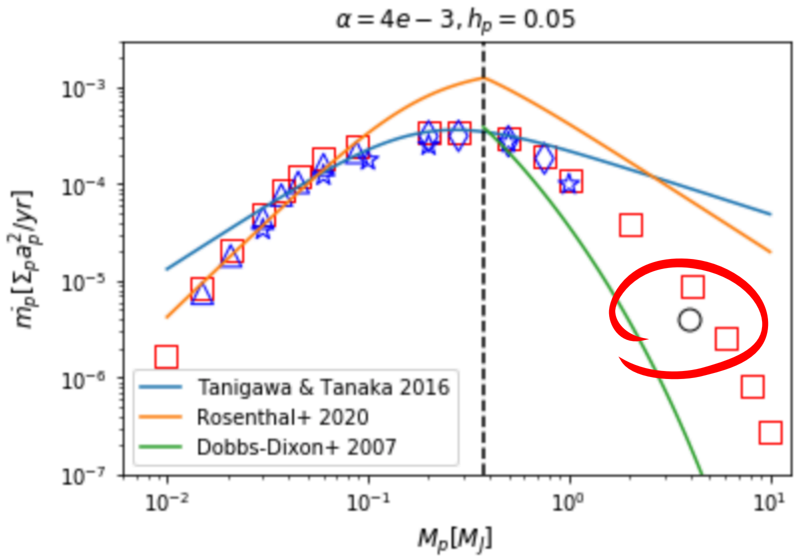
Why no rise of accretion rate up to 10 MJ in the 3D simulation of Bodenheimer+ 2013?

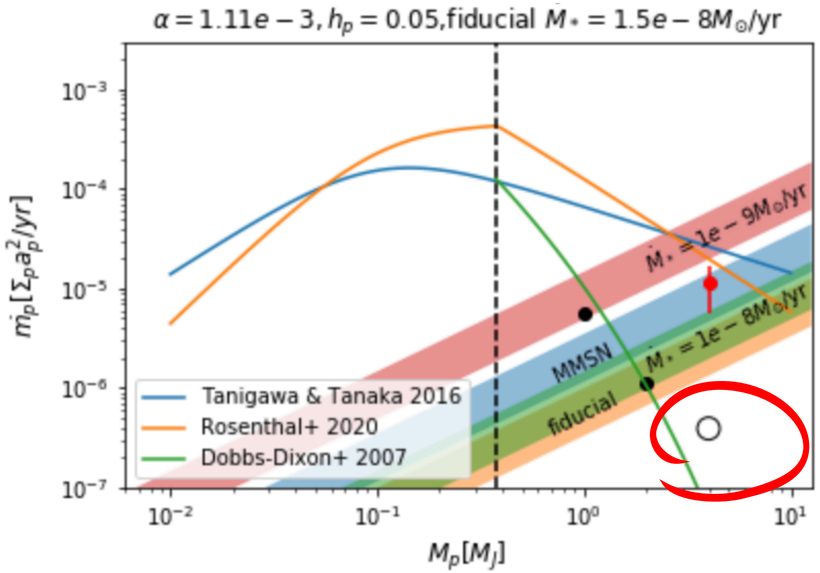
Outstanding Issue: 2D VS 3D

- Low resolution for global streamlines?
- Short simulation time?

Two additional 3D global high res runs







Summary

- Previous estimates and numerical results

  Over-produces ~10MJ planets, and final mass depends sensitively on disk mass
- Effect of the Tidal Barrier

  Some streamlines enter R_H and gets deflected, shrinks cross section
- Dynamical growth with large disk eccentricities in 2D
- Summary of numerical results of accretion rates

  Tidal barrier effective in low-vis scenarios, could constrain final mass before eccentricity growth
- Outstanding Issue: 3D & 2D discrepancy

  No eccentricity excitation in 3D observed