Spark Project

Dust Diffusion in Protostellar Disks and its Effect on Planet Formation

Yixian Chen
Instructor: Prof. Douglas Lin, UCSC

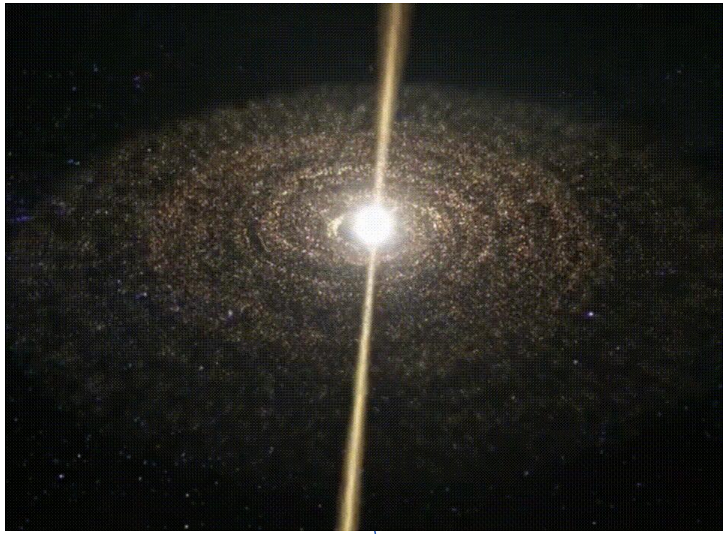
Dep of Physics, Tsinghua University

清華大學
Tsinghua University

PART 1:
Dust Diffusion in Protostellar Disks

Background

Protostellar Disk（原恒星盘）



Composition:99% gas 1% dust
Timescale:1-10 mil yrs

Standard Process of Planet Formation:

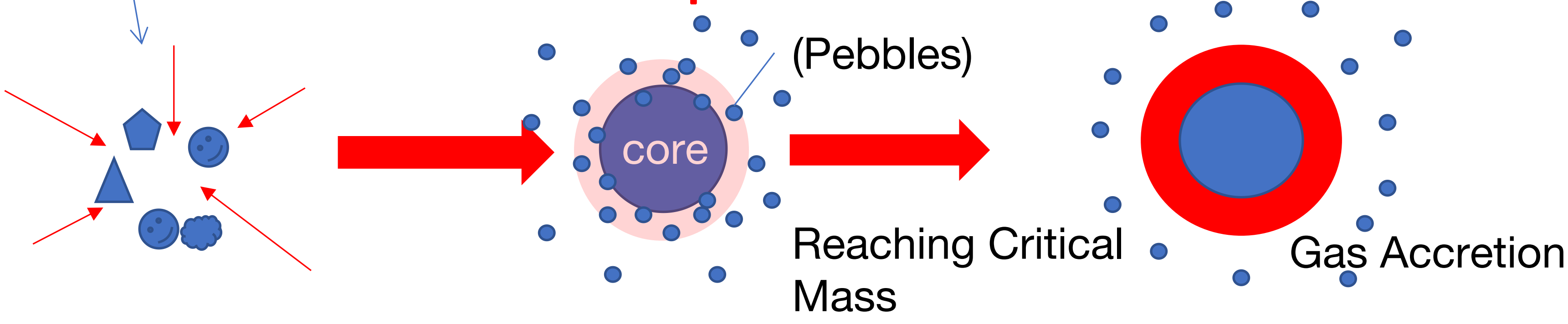
- 1. Accretion (吸积) of pebbles(cm) & planetesimals(km) into a SOLID CORE
- 2. Accretion of GAS envelope

Planets:

- Ice Giant（冰巨星）
- Terrestrial planets（类地）

- Gas Giant (巨行星)
- Super Earth（超级地球）

(10-30 $M_\oplus$,0.1-1AU)



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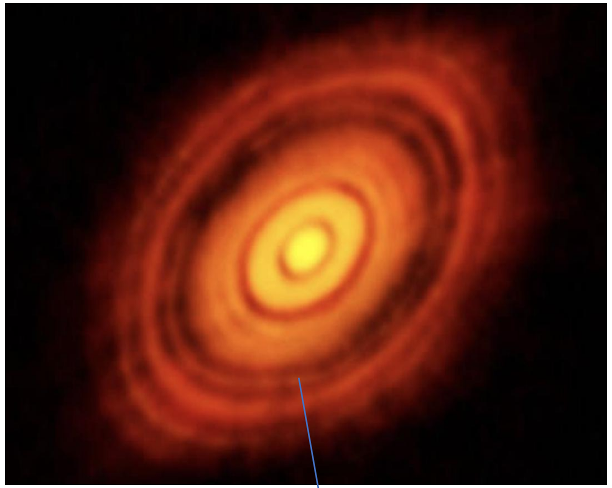

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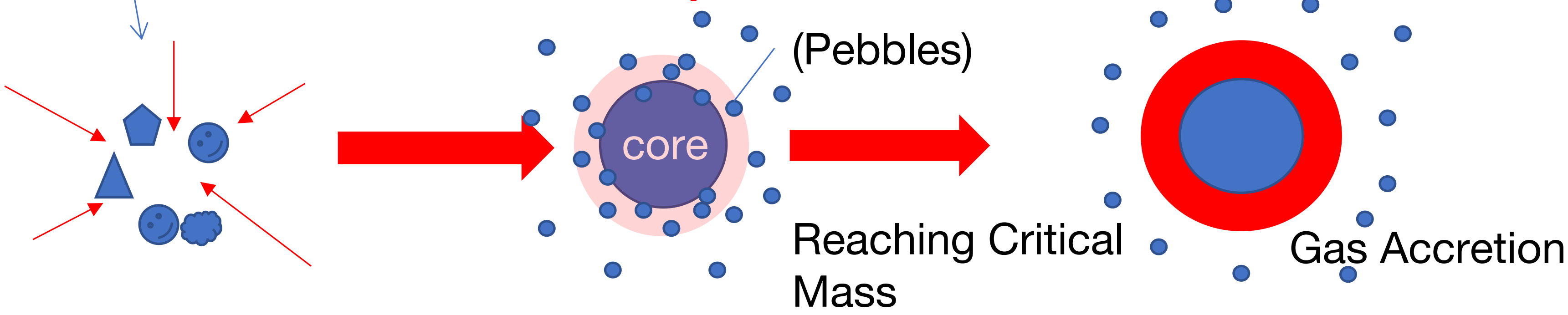
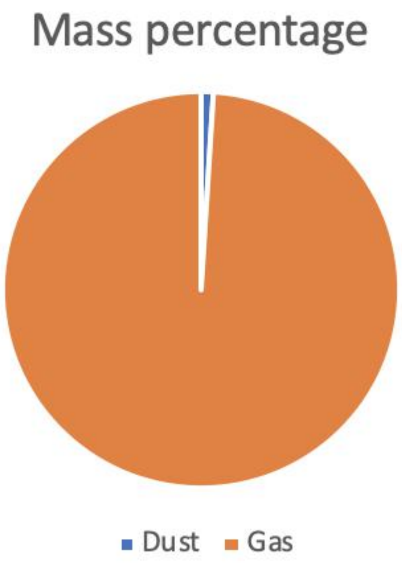
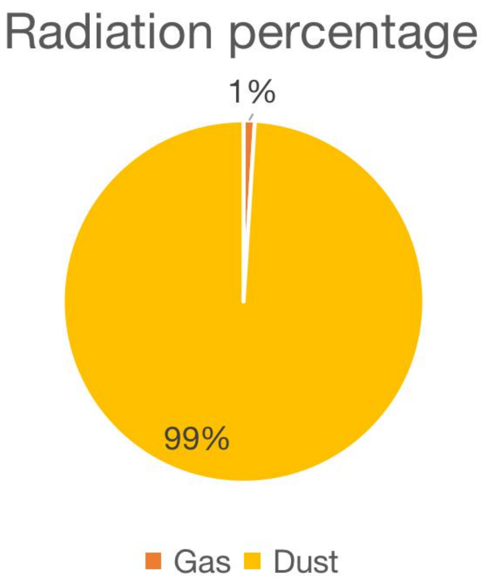
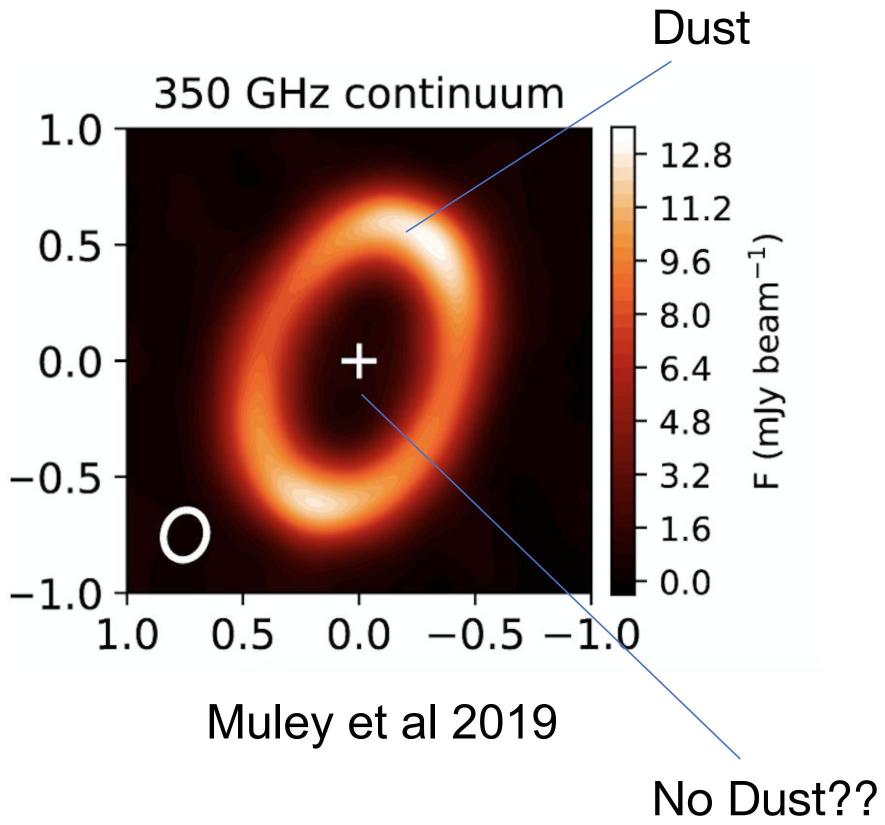


Pebble Isolation Observed

Transition Disks







Muley et al 2019

Dust Profile "Emptied":

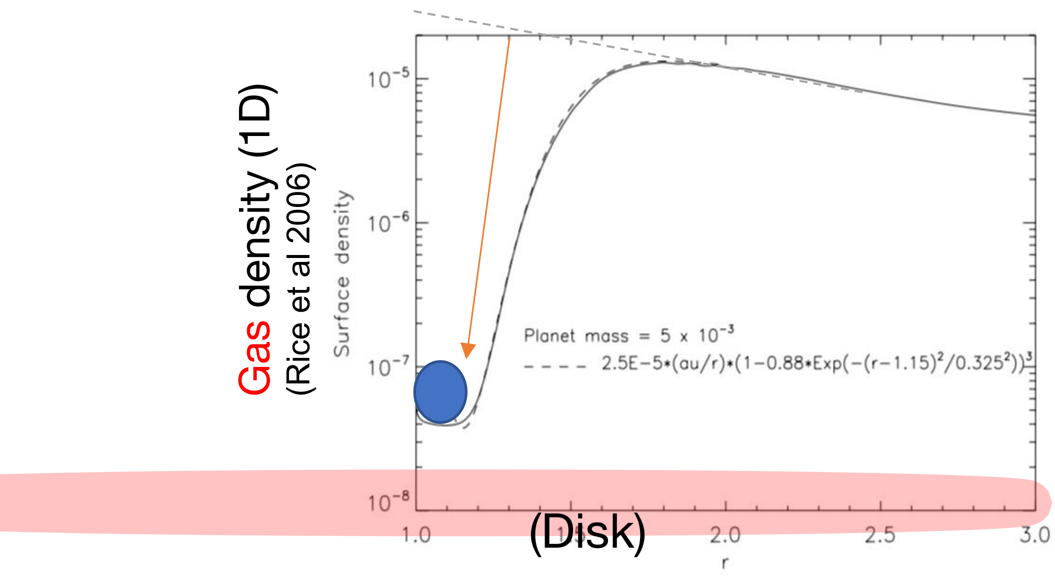
Some of them appears to be entirely devoid of circumstellar material within a certain radius of the star, arguably due to planet formation.

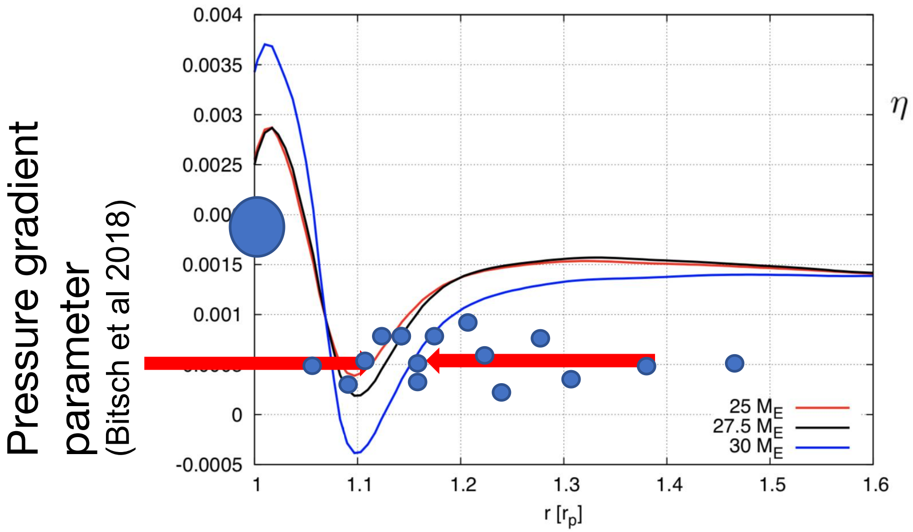
Pebble Isolation: General Picture

Gap Opening in Gas (Rice et al 2006, Bitsch et al 2018)

Momentum Eqn of Gas: $$\frac{v_\phi^2}{r} = \frac{V_K^2}{r} + \frac{1}{\rho}\frac{dP}{dr}$$

The gas velocity is not strictly Keplerian!





$$\eta = -\frac{h^2 \Omega_k r}{2} \frac{ln\rho}{\ln r}$$

$$\propto -\frac{\ln P}{\ln r}$$

if adiabatic

$$\begin{cases} \frac{d\ln\rho}{d\ln r} < 0, \eta > 0, v_\phi < v_K \\ \frac{d\ln\rho}{d\ln r} > 0, \eta < 0, v_\phi > v_K \end{cases}$$

Acting through drag force

Drags down the velocity of dust, which loses angular momentum and spirals inwards

Speeds up dust, which is expelled outwards TOWARDS THE MAXIMA!

Quantify: Contaminant Diffusion (Clarke & Pringle 1988)

Diffusion (扩散) Equation

$$\frac{\partial \Sigma}{\partial t} + \mathrm{div}(\Sigma \mathbf{u}) = 0$$

$$\frac{\partial \sigma}{\partial t} + \mathrm{div}(\sigma \mathbf{u} - \kappa \Sigma \nabla \frac{\sigma}{\Sigma}) = 0$$

$$C := \frac{\sigma}{\Sigma}$$

Dust to Gas density ratio/concentration

$$\Sigma\left(\frac{\partial C}{\partial t} + \mathbf{u}\cdot\nabla C\right) = \mathrm{div}(\kappa \Sigma \nabla C)$$

Axisymmetrical(轴对称薄盘): everything is a function of R

Ratio of diffusion coef over viscosity(气体粘滞系数) $\zeta = \frac{\kappa(R)}{\nu(R)} = constant$

$$\Sigma\frac{\partial C}{\partial t} + \Sigma v_R \frac{\partial C}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\zeta\nu\Sigma\frac{\partial C}{\partial R}\right)$$

Analytical results with NO PLANET PERTURBATION

Clarke & Pringle 1988

Steady Accretion Disk

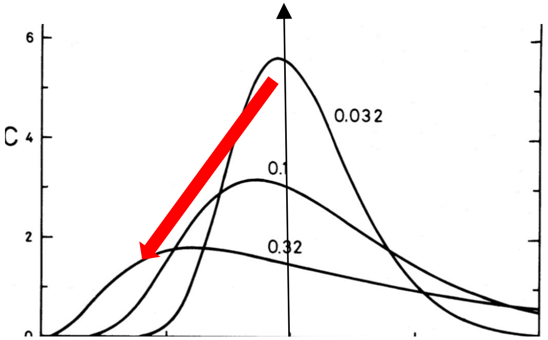
$$\Sigma = \Sigma_0 R^{-a}$$

$$v_R = -\frac{3\nu}{2R} = -\frac{\dot{M}}{2\pi\Sigma_0 R^{1-a}}.$$

For given condition:

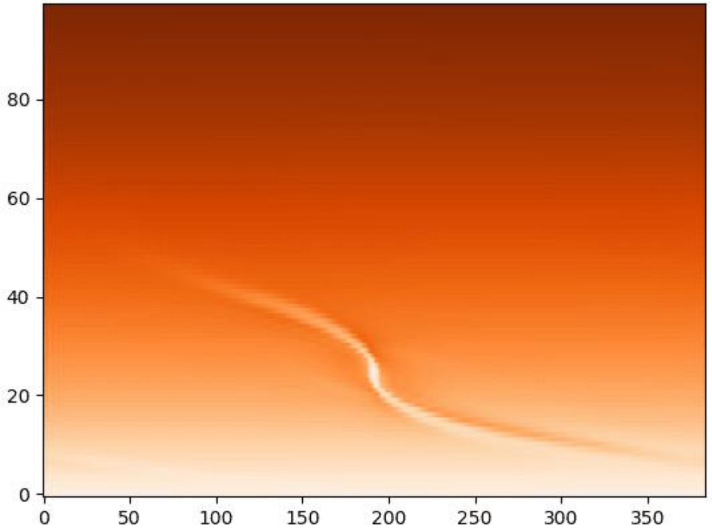
$$C(R,t)|_{t=0} = C_0\delta(R - R_0)$$

$$C(R,t)|_{R=R_{min},R=R_{max}} = 0$$



Planet → Gas(2D)

FARGO3D



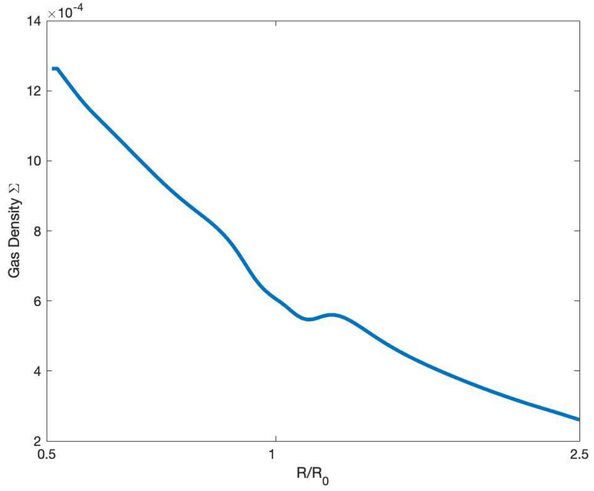
Radial cross-section

Chen & Lin *in prep*

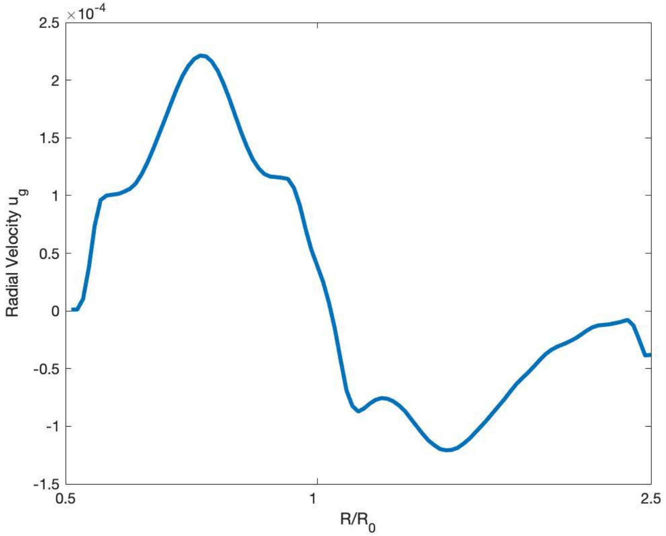
Parameters:

Default unit system:
central star mass=1
orbital radius=1
G=1
Planet=0.001(a planet core)

1. h_0: 0.05
2. Σ_0: 6.3661977237e-4
3. ν_0: 1.0e-5
4. α: 1.0
5. β: 0.25



Gas Density



Radial Velocity

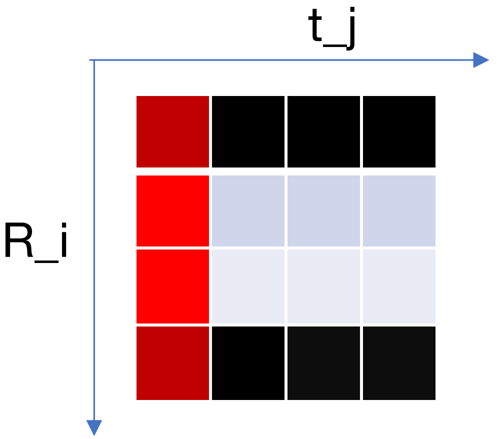
Numerical Method

Jacobi Iteration with MATLAB

$C(R,t) = C(R_i, t_j)$

$$\frac{\partial C}{\partial R}_{i,j} \approx \frac{C(R_{i+1}, t_j) - C(R_{i-1}, t_j)}{\Delta R}$$

$$\frac{\partial C}{\partial t}_{i,j} = \frac{C(R_i, t_j) - C(R_i, t_{j-1})}{\Delta t}$$



Boundary condition

Initial condition

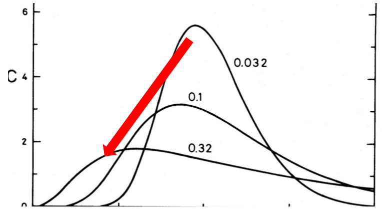
Boundary condition

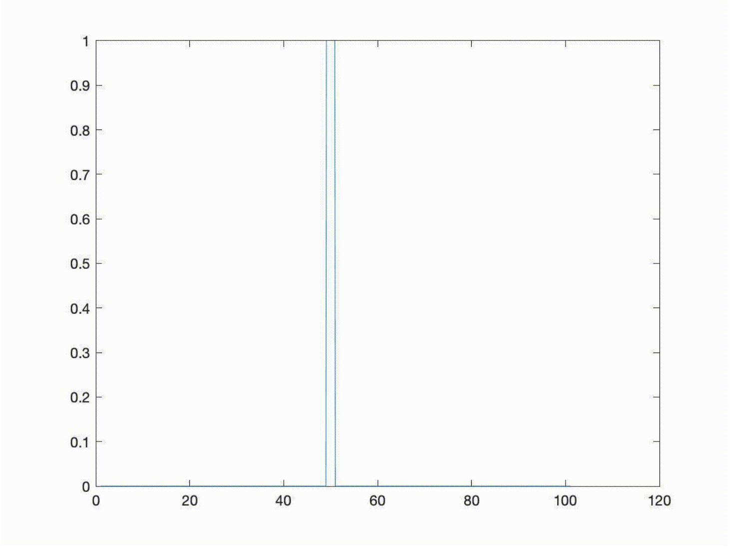
$$C_{i,j} = f(C_{i+1,j}, C_{i-1,j}, C_{i,j-1})$$

Continue to iterate until the C matrix becomes stable!

Method Test

Green function initial

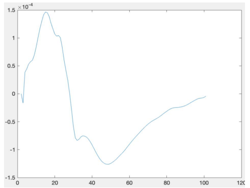




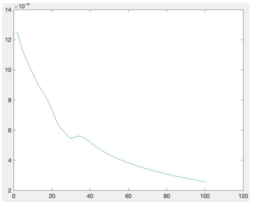
Gas → Dust

$$\Sigma \frac{\partial C}{\partial t} + \Sigma v_{\mathrm{R}} \frac{\partial C}{\partial R} = \frac{1}{R} \frac{\partial}{\partial R} \left(R \zeta \nu \Sigma \frac{\partial C}{\partial R} \right) \longrightarrow \frac{\partial C}{\partial t} + v_{\mathrm{R}}(R) \frac{\partial C}{\partial R} = \kappa_0 [(2 + \frac{\partial \ln \Sigma}{\partial \ln R}(R)) \frac{\partial C}{\partial R} + R \frac{\partial^2 C}{\partial R^2}]$$

Modified Gas Density



Modified Gas Velocity



(Extracted azimuthal mean 1-D data from the 2-D simulation)

For given condition:

$$\sigma(R,t)|_{t=0} = 0.0001 \qquad C(R,t)|_{t=0} = \frac{0.0001}{\Sigma(R)}$$

$$\partial_R C(R,t)|_{R=R_{min}, R=R_{max}} = 0$$

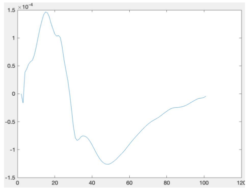
Results:

Chen & Lin *in prep*

Gas → Dust

$$\Sigma\frac{\partial C}{\partial t}+\Sigma v_{\rm R}\frac{\partial C}{\partial R}=\frac{1}{R}\frac{\partial}{\partial R}\left(R\zeta\nu\Sigma\frac{\partial C}{\partial R}\right) \longrightarrow \frac{\partial C}{\partial t}+v_{\rm R}(R)\frac{\partial C}{\partial R}=\kappa_0[(2+\frac{\partial\ln\Sigma}{\partial\ln R}(R))\frac{\partial C}{\partial R}+R\frac{\partial^2 C}{\partial R^2}]$$

Modified Gas Density



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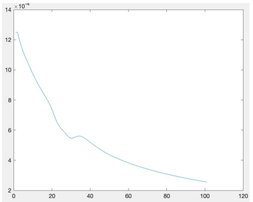

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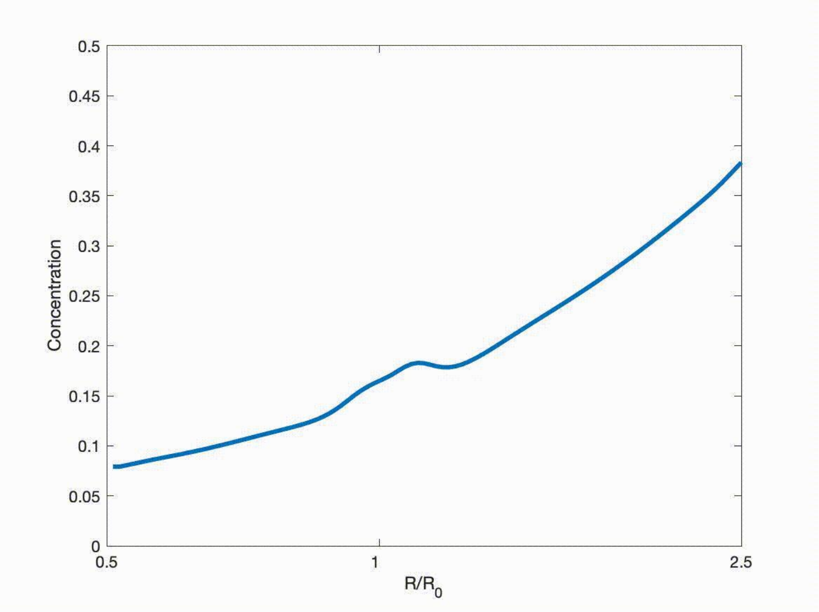
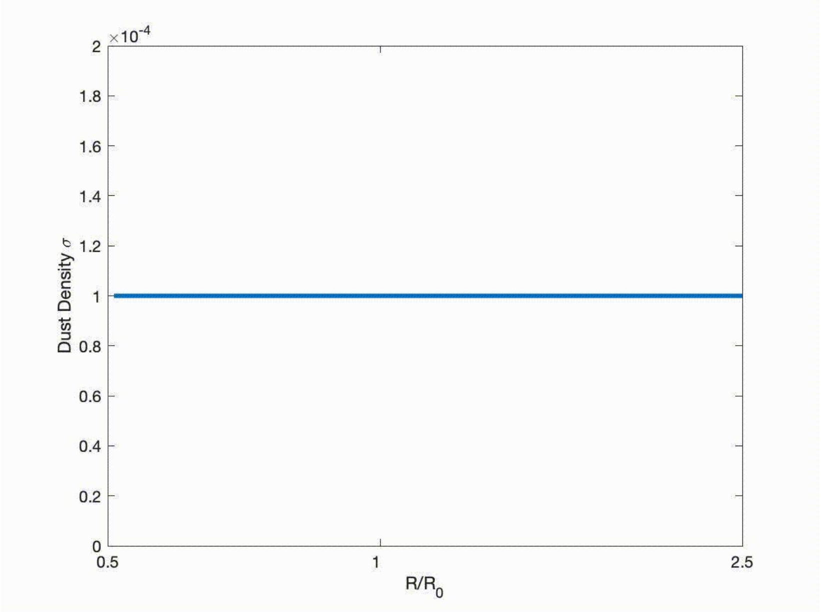
Results: Chen & Lin *in prep*





Modification (+ Gas Drag)

v: relative azimuthal velocity; u: radial velocity

$$\begin{cases} -2v_p\Omega_k = \dfrac{u_g - u_p}{\tau_s} \\ \dfrac{1}{2}u_p\Omega_k = \dfrac{v_g - v_p}{\tau_s} \\ -2v_g\Omega_k = -\dfrac{\rho_p}{\rho}\dfrac{u_g - u_p}{\tau_s} + 2\eta\Omega_k \\ \dfrac{1}{2}u_g\Omega_k = -\dfrac{\rho_p}{\rho}\dfrac{v_g - v_p}{\tau_s} + \dfrac{1}{2}\xi\Omega_K \end{cases}$$

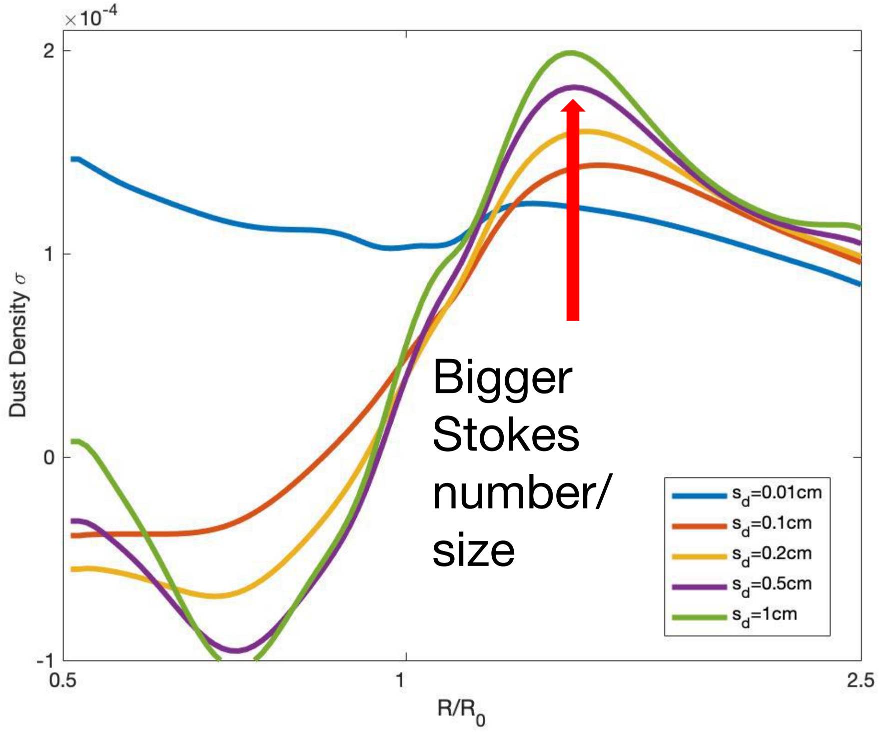
$$\eta = -\frac{h^2\Omega_k r}{2}\frac{ln\rho}{\ln r}$$

$$\frac{1}{2}\xi = \frac{\partial}{pr^2\Omega_K\partial r}\left(\rho\nu r^3\frac{\partial\Omega_k}{\partial r}\right)$$

Affiliated with relaxation time(弛豫时间).
Eqn 1,2 ----Gas drag
Eqn 3,4 ---- Feedback

$$\begin{cases} \frac{1}{2}u_g\Omega_k = \frac{v_{gas}-v_g}{\tau} \\ -2v_g\Omega_k = \frac{u_{gas}-u_g}{\tau} \end{cases} \Rightarrow u_g = \frac{u_{gas}}{1+(\tau\Omega_k)^2}$$

Stokes number, proportional to dust size s_d



Conclusion:
Bigger dust grains (pebbles) are more likely to be totally blocked.(PEBBLE ISOLATION)

Flattening (+Gas Drag +Feedback)

$$\begin{cases} \frac{1}{2}u\Omega_k = -\frac{\rho_p}{\rho}\frac{v-v_p}{\tau_s} + \frac{1}{2}\xi\Omega_K \\ \frac{1}{2}u_p\Omega_k = \frac{v-v_p}{\tau_s} \\ -2v_p\Omega_k = \frac{u-u_p}{\tau_s} \\ -2v\Omega_k = -\frac{\rho_p}{\rho}\frac{u-u_p}{\tau_s} + 2\eta\Omega_k \end{cases}$$

$\Delta v := v - v_p, \Delta u := u - u_p$ →

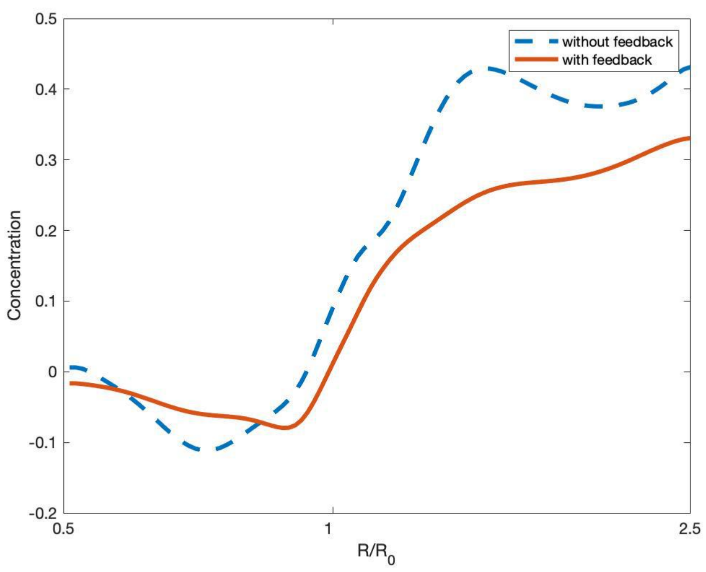
$$\begin{cases} -\Delta v = \eta - \frac{1}{2}(1+C)\frac{\Delta u}{S_t} = 0 \\ \Delta u = \xi - 2(1+C)\frac{\Delta v}{S_t} = u_g \end{cases}$$

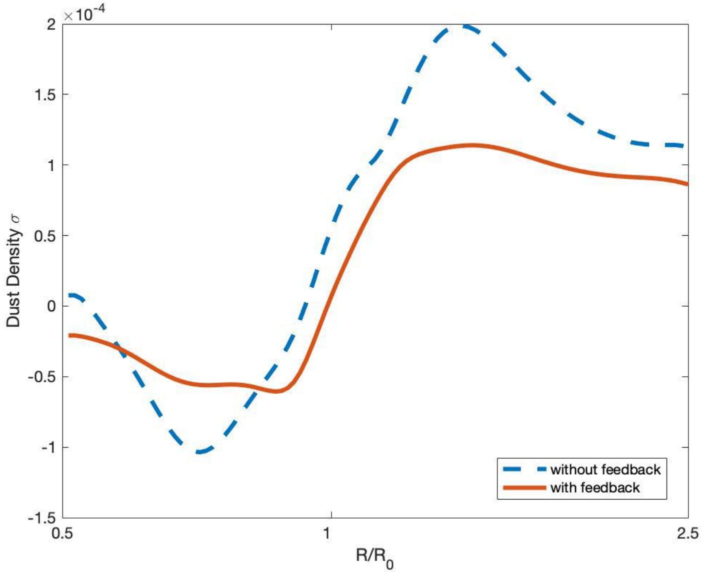
Critical Concentration $C = \frac{2\eta S_t}{u_g} - 1$

After reaching critical concentration, the dust begins to move outwards and gain a positive radial velocity, to accumulate elsewhere

To approximate, we just let the radial velocity of dust to gradually reduce to 0:

Result: much flatter than with no feedback





PART 2:
Implication on Planet Formation

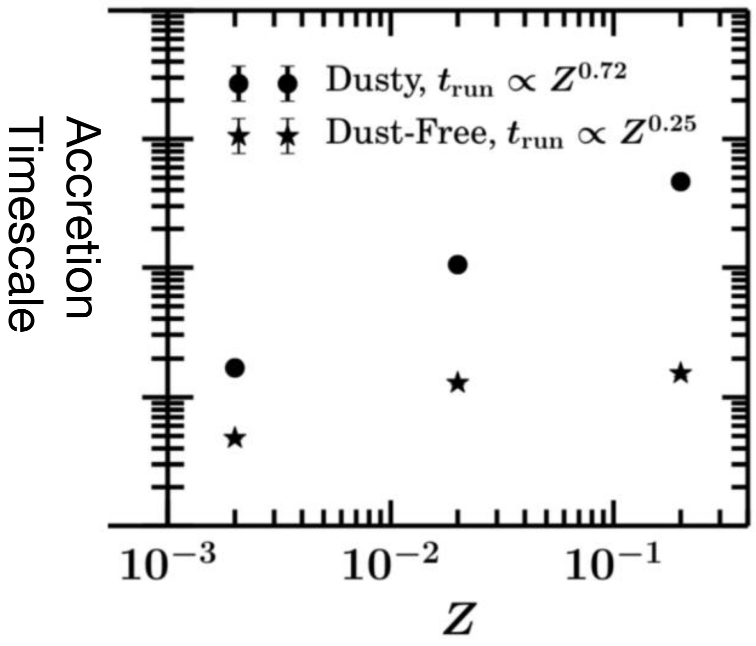
Opacity(不透明度)

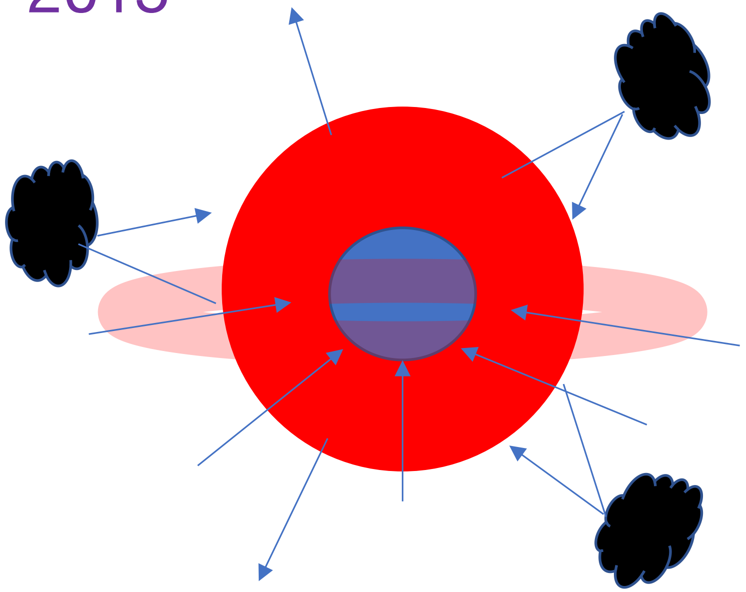
$$\kappa_{\rm rcb} = \kappa_0(\rho_{\rm rcb}/\rho_0)^\alpha(T_{\rm rcb}/T_0)^\beta(Z/Z_0)^\delta$$

Contributed by dust grains and metallicity!

- Grain/Metal Contaminant↓
- Opacity ↓
- Cooling↑
- Accretion↑

"To Cool is to Accrete!"
——Lee & Chiang 2015





Opacity in Pit and Pileup

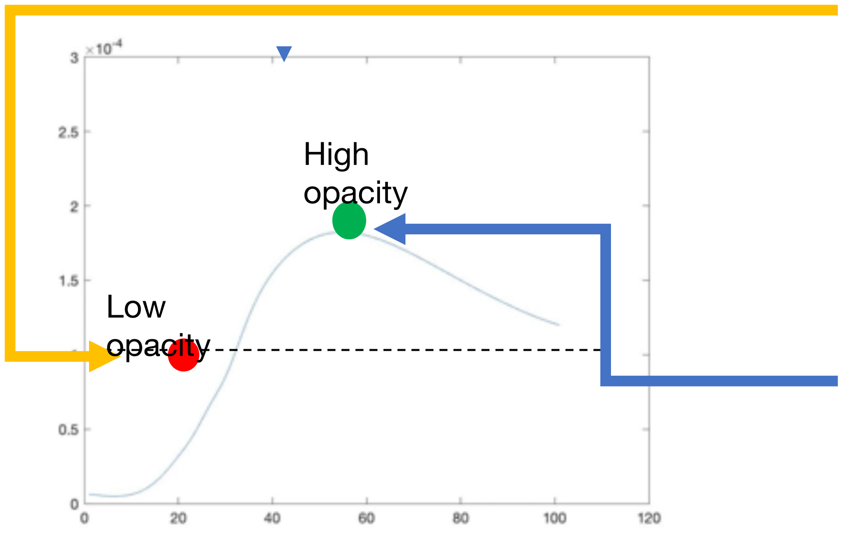
Creationist's Point of View

Opacity/Metallicity is a pre-set **CONSTANT**!

Low Metallicity/Opacity -> Quick Accretion, Short Timescale

High Metallicity/Opacity -> Slow Accretion, Long Timescale

Evolutionist's Point of View



Density profile

PIT:

Accreting planets have the ability to lower the original dust density around the vicinity by generating dust barriers

enhancing its own cooling and accretion

Hard to form a Super-Earth by itself (unless anomolies)

PILEUP（堆积物）：

A NEW core's (once formed) accretion would be hindered by the opacity accumulated under the influence of the first planet

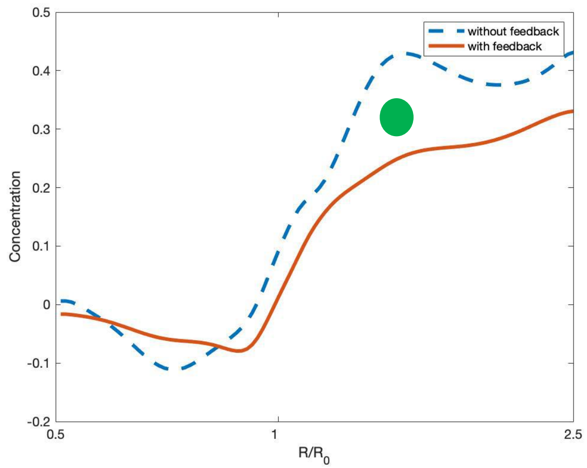
Might remain a Super-Earth

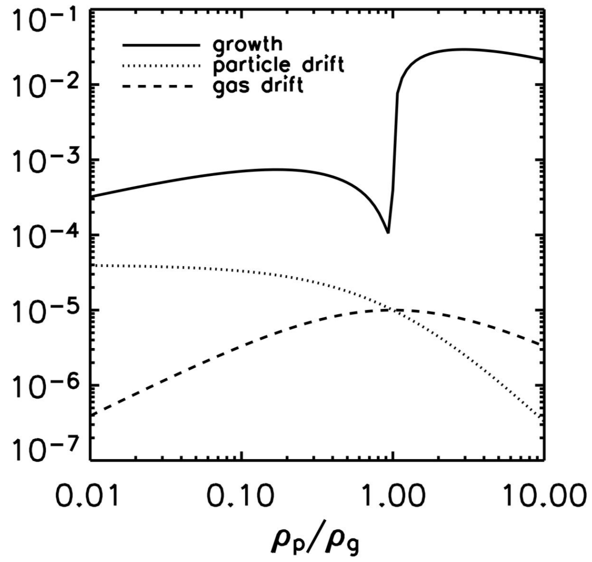
Core Formation in Pileup

Streaming Instability (Youdin & Goodman 2005)

In places where C~1, the interaction between GAS and DUST gives rise to rapid growth of PLANETISIMAL (CORE)

$$C = \frac{2\eta S_t}{u_g} - 1$$



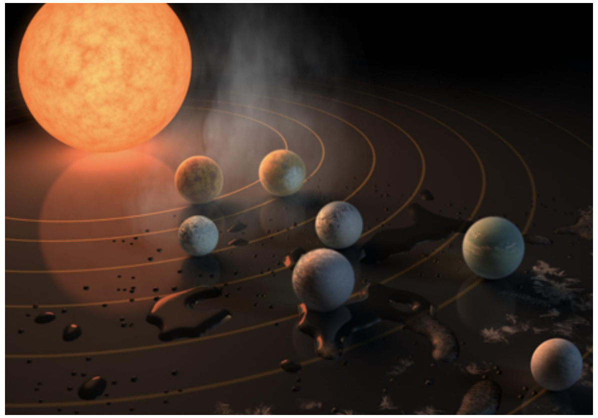


C_crit~1: dust accumulates enough to form planetesimals and cores

C_crit<<1: reaches a ceiling and flattens out before formation

A Chain Reaction?

cores form out of the pileup one after another and push the pileup further back->a string of superearths



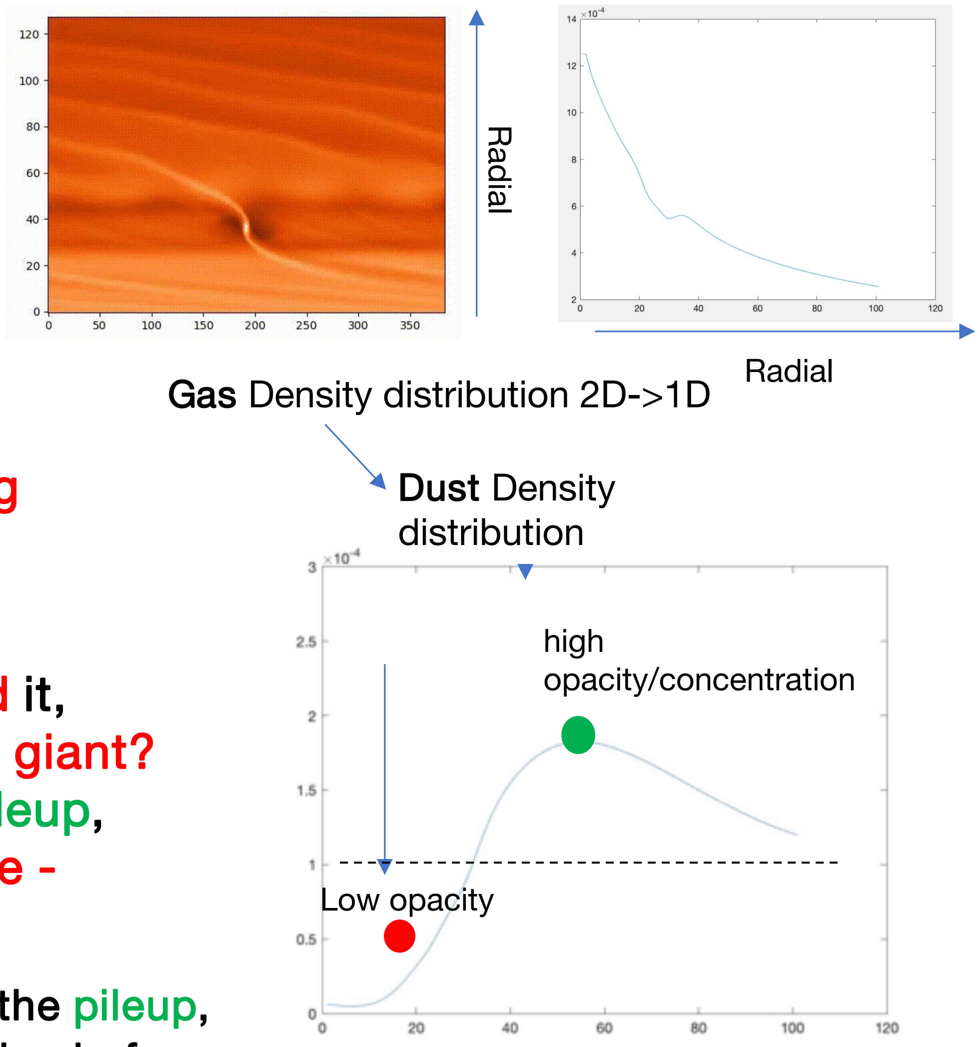
TRAPPIST-1 system

Summary

PART 1
Dust Diffusion in Protostellar disks

- Planet formation
- Gas Gapping and Pebble Isolation
  (Explained with Diffusion Equation)

Conclusion: Accreting planets have the ability to change the dust density profile according to particle size around its vicinity by generating dust barriers
Features1)Bigger dust gets blocked more 2)Flattening



PART 2
Effects on Planet Formation

- Opacity
  - Lower opacity/dust density around it, enhances its own accretion -> gas giant?
  - High opacity/dust density in the pileup, hinders the accretion of a new core ->super earth?
- **Core formation:** Generates instability/core forming in the pileup, leading to forming of a new core if the Critical Concentration before “flattening” reaches ~1, potentials of sequential formation （接连形成）